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RESEARCH ARTICLE - MATHEMATICS

Jordan Generalized Higher Symmetric Reverse Bi-Derivations on Prime Γ-Rings

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Abstract

In this paper we show that, every Jordan generalized higher symmetric reverse bi-derivations is a generalized higher symmetric reverse bi-derivations on a Γ −ring M. Additionally, a generalized higher symmetric reverse is Jordan generalized higher symmetric triple reverse bi-derivation on a 2-torsion free Γ −ring M.

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1. Introduction

Given two additive abelian groups, M and Γ, if the following requirements hold for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$, then M is said to be $\Gamma-$ ring

1. $a\alpha b \in M$

2. $a\alpha(b+c) = a\alpha b + a\alpha c,\ \ a(\alpha+\beta)b = a\alpha b + a\beta b,\ \ (a+b)\alpha c = a\alpha c + b\alpha c$

3. $(a\alpha b)\beta c = a\alpha(b\beta c)$

N. Nobusawa 1964 [1] proposed the concept of Γ-ring for the first time, and W.E. Barnes 1966 [2]expanded on it as stated in the definition above. All rings are known to be Γ-ring. According to S. Kyuno in [3], M is referred to be prime if aΓMΓb=(0) implies that a=0 or b=0, and its called semiprime if aΓMΓa=(0) implies that a=0 for each a,b∈M, M is named if n-torsion free if na =0 for all a∈M means that a=0 where n is positive integer. A derivation of F.J. Jing on Γ-ring is defined in [4]. A generalized derivation on M is defined by Y. Ceven and M.A. Ozturk in [5]. A Jordan derivation on M is defined by M. Sapanci and A. Nakajima in [6]. A symmetric bi-derivation on M is introduced to M.A. Ozturk in [7]. The idea presented in [8] by regarding a reverse derivation on M. A higher bi-derivation is defined by S.M. Salih and A.M. Marir in [9].

S. M. Salih and A. M. Marir introduced the ideas of generalized higher bi-derivation ,Jordan generalized higher bi-derivation and Jordan triple generalized higher bi-derivation on M in [10]. In [11], a generalized symmetric higher bi-derivation on M is defined by S. J. Shaker.

In this research, we demonstrate that, for our concepts of a generalized higher symmetric reverse bi-derivations, Jordan generalized higher symmetric reverse bi-derivations and Jordan generalized higher symmetric triple reverse bi-derivations on a prim Γ −rings M.2.

2. Generalized Higher Symmetric Reverse Bi-Derivations on Prime Γ-Rings

This section will provide an overview of the concept of generalized higher symmetric reverse bi-derivations as well as various inductance-related lemmas that will aid in the proof of several theorems.

Definition 2.1

Let $D = (D_i)_{i\in N}$ as a family of symmetric bi-additive mappings within a prime Γ-rings $M \times M$ into M, then D is known to as generalized higher symmetric reverse bi-derivations on M if there exists $d = (d_i)_{i\in N}$ higher symmetric reverse bi-derivations on M×M into M, then for all $w, v, t \in M$, α∈ Γ and n ∈N

$$D_n(w\alpha v, t) = \sum_{i+j=n} D_i(v, t)\alpha d_j(w, t)$$

Example 2.2

M is a Γ-ring if it is a ring of all 2×2 matrices of integer values with M=$\left\{\begin{pmatrix} 0 & 0 \\ w & 0 \end{pmatrix}; w \in Z\right\}$ and $\Gamma = \left\{\begin{pmatrix} \sigma & 0 \\ 0 & 0 \end{pmatrix}; \sigma \in Z\right\}$. Let $D_n: M \times M \longrightarrow M$ be a symmetric bi-additive mappings of M with $D_n\left(\begin{pmatrix} 0 & 0 \\ w & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ v & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 \\ (n^3 wv & 0 \end{pmatrix}$ for everyone $\begin{pmatrix} 0 & 0 \\ w & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ v & 0 \end{pmatrix} \in M$ and n∈N. Then there exist a higher symmetric reverse bi-derivations defined as follows: let $d_n: M \times M \longrightarrow M$ be a symmetric bi-additive mappings of a Γ-ring M such that $d_n\left(\begin{pmatrix} 0 & 0 \\ w & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ v & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 \\ (n+4)wv & 0 \end{pmatrix}$ for all $\begin{pmatrix} 0 & 0 \\ w & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ v & 0 \end{pmatrix} \in M$ and $n \in N$. Therefore D_n is a generalized higher symmetric reverse bi-derivations on M.

Example 2.3

Assume that $M = \{\begin{bmatrix} w & v \\ t & r \end{bmatrix}; w, v, t, r \in Z\}$ and $\Gamma = \{\binom{\alpha}{0}; \alpha \in Z\}$. M is therefore Γ-ring. Let $G = (G_i)_{i\in N}$ represent a family of generalized higher symmetric reverse bi-derivations on M. Then, $d = (d_i)_{i\in n}$ of M is a higher symmetric reverse bi-derivations. If we define $D = (D_i)_{i\in N}$ be a family of symmetric bi-additive mappings of $M \times M$ into M where

$$D_n\left(\begin{bmatrix} w & v \\ t & r \end{bmatrix}, \begin{bmatrix} e & f \\ g & h \end{bmatrix}\right) = (d_n(\begin{bmatrix} w & v \\ t & r \end{bmatrix}) \qquad d_n\left(\begin{bmatrix} e & f \\ g & h \end{bmatrix}\right))$$

Consider a family of symmetric bi-additive mappings from M×M into M, $F = (F_i)_{i\in N}$ denoted by

$$F_n\left(\begin{bmatrix} w & v \\ t & r \end{bmatrix}, \begin{bmatrix} e & f \\ g & h \end{bmatrix}\right) = (G_n(\begin{bmatrix} w & v \\ t & r \end{bmatrix}) \qquad G_n\left(\begin{bmatrix} e & f \\ g & h \end{bmatrix}\right))$$

Hence F represents generalized higher symmetric reverse bi-derivations on M.

Definition 2.4

Considering a prime Γ-ring M with a family of symmetric bi-additive maps $D = (D_i)_{i\in N}$ from $M \times M$ into M, D is referred to as Jordan generalized higher symmetric reverse bi-derivations, if there exists $d = (d_i)_{i\in N}$ Jordan higher symmetric reverse bi-derivations on M, such that for all $w, t \in M$, $\alpha \in \Gamma$ and n ∈N

$$D_n(w\alpha w, t) = \sum_{i+j=n} D_i(w, t)\alpha d_j(w, t)$$

Example 2.5

Given a Γ–ring M and a ∈ M, let $w\Gamma a\Gamma w$=(0) ∀ w ∈ M and aΓa=(0). Let d be a set of symmetric bi-additive maps from M×M into M with $d_n(w, v)$= $nw\alpha a + a\alpha w$ for each of $w, v \in M$ and $\alpha \in \Gamma$ and for each n ∈ N. Assume that $B = (B_i)_{i\in N}$ is a set of symmetric bi-additive mappings of M where, with any n ∈ N, $B_n(w, v)$ =$nw\alpha a$ for all $w, v, a \in M$ and α ∈ Γ. B is a Jordan generalized higher symmetric reverse bi-derivations on M, but it is not a generalized higher symmetric reverse bi-derivations on M.

Definition 2. 6

Assume that $D = (D_i)_{i\in N}$ is a set of symmetric bi-additive mappings from a prime Γ-ring M×M into M. In this case, D is referred to as a Jordan generalized higher symmetric triple reverse bi-derivations on M, if there exists a Jordan higher symmetric triple reverse bi-derivations $d = (d_i)_{i\in N}$ from M×M into M, where, for every $w, v, r \in M$, α, β ∈ Γ and n∈N then

$$D_n(w\alpha v\beta w, r) = \sum_{i+j+k=n} D_i(w, r)\alpha d_j(v, r)\beta\, d_k(w, r)$$

Example 2.7

We know that D_n and F_n both Jordan generalized higher symmetric triple reverse bi-derivations on M from Examples (2.2) and (2.3).

Lemma 2.8

For any of them $w, v, t, r \in M$ and $\alpha, \beta \in \Gamma$, if $D = (D_i)_{i\in N}$ is a family of Jordan generalized higher symmetric reverse bi-derivations on a Γ-ring M×M into M, then for all $w, v, t, r \in M$ and $\alpha, \beta \in \Gamma$, then the following is hold.

(i) $D_n(w\alpha v + v\alpha w, t) = \sum_{i+j=n} D_i(v, t)\alpha d_j(w, t) + D_i(w, t)\alpha d_j(v, t)$

(ii) $D_n(w\alpha v\beta t + t\alpha v\beta w, r)$

$$= \sum_{i+j+k=n} D_i(t, r)\beta d_j(v, r)\alpha d_k(w, r) + D_i(w, r)\beta d_j(v, r)\alpha d_k(t, r)$$

(iii) Consider M to be a commutative 2-torsion free Γ-ring, then

$$D_n(w\alpha v\beta t, r) = \sum_{i+j+k=n} D_i(t, r)\alpha d_j(v, r)\beta d_k(w, r)$$

Proof (i)

In definition (2.4), substitute w for $w + v$, we get

$$D_n((w + v)\alpha(w + v), t) = \sum_{i+j=n} D_i(w + v, t)\alpha d_j(w + v, t)$$

$$= \sum_{i+j=n}(D_i(w, t) + D_i(v, t))\alpha(d_j(w, t) + d_j(v, t))$$

$=\sum_{i+j=n} D_i(w,t)\alpha d_j(w,t) + D_i(w,t)\alpha d_j(v,t) + D_i(v,t)\alpha d_j(w,t)+D_i(v,t)\alpha d_j(v,t)$ (1)

Furthermore,

$$D_n((w+v)\alpha(w+v),t) = D_n(w\alpha w + w\alpha v + v\alpha w + v\alpha v,t)$$
$$= D_n(w\alpha w + v\alpha v,t) + D_n(w\alpha v + v\alpha w,t)$$
$$= D_n(w\alpha w,t) + D_n(v\alpha v,t) + D_n(w\alpha v + v\alpha w,t)$$

$= \sum_{i+j=n} D_i(w,t)\alpha d_j(w,t) + D_i(v,t)\alpha d_j(v,t) + D_n(w\alpha v + v\alpha w,t)$ (2)

When we compare (1) and (2), we obtain

$$D_n(w\alpha v + v\alpha w,t) = \sum_{i+j=n} D_i(v,t)\alpha d_j(w,t) + D_i(w,t)\alpha d_j(v,t)$$

Prooe (ii)

When we change w in definition (2.6) to $w+v$, we obtain

$D_n((w+t)\alpha v\beta(w+t),r)$

$$= \sum_{i+j+k=n} D_i(w+t,r)\alpha d_j(v,r)\beta d_k(w+t,r)$$

$$= \sum_{i+j+k=n} (D_i(w,r)+D_i(t,r))\alpha d_j(v,r)\beta(d_k(w,r)+d_k(t,r))$$

$=\sum_{i+j+k=n} D_i(w,r)\alpha d_j(v,r)\beta d_k(w,r) + D_i(w,r)\alpha d_j(v,r)\beta d_k(t,r) +$
$D_i(t,r)\alpha d_j(v,r)\beta d_k(w,r) + D_i(t,r)\alpha d_j(v,r)\beta d_k(t,r)$ (3)

Furthermore,

$D_n((w+t)\alpha v\beta(w+t),r) = D_n(w\alpha v\beta w + w\alpha v\beta t + t\alpha v\beta w + t\alpha v\beta t,r)$

$= D_n(w\alpha v\beta w + t\alpha v\beta t,r) + D_n(w\alpha v\beta t + t\alpha v\beta w,r)$

$=\sum_{i+j+k=n} D_i(w,r)\alpha d_j(v,r)\beta d_k(w,r) + D_i(t,r)\alpha d_j(v,r)\beta d_k(t,r) + D_n(w\alpha v\beta t +$
$t\alpha v\beta w,r)$ (4)

When we compare (3) and (4), we obtain the needed result.

Proof (iii)

By ii and since $w\alpha v\beta t = t\alpha v\beta w$ for everyone $w,v,t \in M$ and $\alpha,\beta \in \Gamma$, we get

$$D_n(2w\alpha v\beta t,r) = 2\sum_{i+j+k=n} D_i(t,r)\alpha d_j(v,r)\beta d_k(w,r)$$

By M is 2-torsion free Γ −ring then

$$D_n(w\alpha v\beta t,r) = \sum_{i+j+k=n} D_i(t,r)d_j(v,r)\beta d_k(w,r)$$

Remark 2.9

Given a prime Γ-ring M and $D = (D_i)_{i\in N}$ as the Jordan generalized higher symmetric reverse bi-derivations on M, for each n ∈N, $w,v,t \in M$ and $\alpha \in \Gamma$, we make clear

$$\mu_n(w,v,t)_\alpha = D_n(w\alpha v,t) - \sum_{i+j=n} D_i(v,t)\,\alpha d_j(w,t)$$

Lemma 2.10

Let $D = (D_i)_{i \in N}$ be Jordan generalized higher symmetric reverse bi-derivation on a Γ −ring M, for every n ∈N, $w, v, t \in$ M and $\alpha \in \Gamma$ then

(i)$\mu_n(w, v, t)_\alpha = -\mu_n(v, w, t)_\alpha$

(ii) $\mu_n((w + v), t, r)_\alpha = \mu_n(w, t, r)_\alpha + \mu_n(v, t, r)_\alpha$

(iii) $\mu_n(w, (v + t), r)_\alpha = \mu_n(w, v, r)_\alpha + \mu_n(w, t, r)_\alpha$

Proof (i)

As d is a symmetric bi-additive mapping on M according to lemma (2.8) (i), it follows that

$$D_n(w\alpha v + v\alpha w, t) = \sum_{i+j=n} D_i(v, t)\alpha d_j(w, t) + D_i(w, t)\alpha d_j(v, t)$$

$$D_n(w\alpha v, t) + D_n(v\alpha w, t) = \sum_{i+j=n} D_i(v, t)\alpha d_j(w, t) + \sum_{i+j=n} D_i(w, t)\alpha d_j(v, t)$$

$$D_n(w\alpha v, t) - \sum_{i+j=n} D_i(v, t)\alpha d_j(w, t) = -D_n(v\alpha w, t) + \sum_{i+j=n} D_i(w, t)\alpha d_j(v, t)$$

$$D_n(w\alpha v, t) - \sum_{i+j=n} D_i(v, t)\alpha d_j(w, t) = -(D_n(v\alpha w, t) - \sum_{i+j=n} D_i(w, t)\alpha d_j(v, t))$$

$$\mu_n(w, v, t)_\alpha = -\mu_n(v, w, t)_\alpha$$

Proof (ii)

$$\mu_n((w + v), t, r)_\alpha = D_n\big((w + v)\alpha t, r\big) - \sum_{i+j=n} D_i(t, r)\alpha\, d_j(w + v, r)$$

$=D_n(w\alpha t + v\alpha t, r) - (\sum_{i+j=n} D_i(t, r)\,\alpha d_j(w, r) + D_i(t, r)\alpha d_j(v, r)$

Since D_n is bi-additive mapping on M, we get

$\mu_n\big((w + v), \alpha, t), r\big) = D_n(w\alpha t, r) - \sum_{i+j=n} D_i(t, r)\,\alpha d_j(w, r) + D_n(v\alpha t, r) - \sum_{i+j=n} D_i(t, r)\alpha d_j(v, r)$

$= \mu_n(w, t, r)_\alpha + \mu_n(v, t, r)_\alpha$

Proof (iii)

In the same way as proof (ii)

Theorem 2. 11

Every Jordan generalized higher symmetric reverse bi-derivation on a Γ −ring M is generalized higher symmetric reverse bi-derivation on M.

Proof

Let $D = (D_i)_{i \in N}$ be a family of Jordan generalized higher symmetric reverse bi-derivations on M.

By lemma (2.8) (i) we have

$$D_n(w\alpha v + v\alpha w, t) = \sum_{i+j=n} D_i(v, t)\alpha d_j(w, t) + D_i(w, t)\alpha d_j(v, t)$$
$$= \sum_{i+j=n} D_i(v, t)\alpha d_j(w, t) + \sum_{i+j=n} D_i(w, t)\alpha d_j(v, t) \quad (5)$$

Furthermore,

Given that D_n is bi-additive mapping

$$D_n(w\alpha v + v\alpha w, t) = D_n(w\alpha v, t) + D_n(v\alpha w, t)$$
$$= D_n(w\alpha v, t) + \sum_{i+j=n} D_i(w, t)\alpha d_j(v, t) \qquad (6)$$

When we compare equations (5) and (6), we obtain

$$D_n(w\alpha v, t) = \sum_{i+j=n} D_i(v, t)\alpha d_j(w, t)$$

Then D is generalized higher symmetric reverse bi-derivation on M.

Proposition 2.12

Every Jordan generalized higher symmetric reverse bi-derivations of 2-torsion free $\Gamma-$ring M is Jordan generalized higher symmetric triple reverse bi-derivations on M such that $w\alpha v\beta w = w\beta v\alpha w$ for every $w, v \in M$ and $\alpha, \beta \in \Gamma$.

Proof

Let $D = (D_i)_{i\in N}$ be Jordan generalized higher symmetric reverse bi-derivations of M.

From lemma 2.8 (i) replacing v by $w\beta v + v\beta w$ we get

$$D_n(w\alpha(w\beta v + v\beta w) + (w\beta v + v\beta w)\alpha w, t)$$
$$= D_n(w\alpha(w\beta v) + w\alpha(v\beta w) + (w\beta v)\alpha w + (v\beta w)\alpha w, t)$$
$$= D_n((w\alpha w)\beta v) + (w\alpha v)\beta w) + (w\beta v)\alpha w + (v\beta w)\alpha w, t)$$
$$= D_n\big((w\alpha w)\beta v, t\big) + D_n((w\alpha v)\beta w, t) + D_n((w\beta v)\alpha w, t) + D_n\big((v\beta w)\alpha w, t\big)$$
$$= \sum_{i+j=n} D_i(v, t)\beta d_j(w\alpha w, t) + D_i(w, t)\beta d_j(w\alpha v, t) + D_i(w, t)\alpha d_j(w\beta v, t)$$
$$+ D_i(w, t)\alpha d_j(v\beta w, t)$$
$$= \sum_{i+j+k=n} D_i(v, t)\beta d_j(w, t)\alpha d_k(w, t) + D_i(w, t)\beta d_j(v, t)\alpha d_k(w, t)$$
$$+ D_i(w, t)\alpha d_j(v, t)\beta d_k(w, t) + D_i(w, t)\alpha d_j(w, t)\beta d_k(v, t))$$
$$= \sum_{i+j+k=n} D_i(v, t)\beta d_j(w, t)\alpha d_k(w, t) + \sum_{i+j+k=n} D_i(w, t)\beta d_j(v, t)\alpha d_k(w, t) +$$
$$\sum_{i+j+k=n} D_i(w, t)\alpha d_j(v, t)\beta d_k(w, t) + \sum_{i+j+k=n} D_i(w, t)\alpha d_j(w, t)\beta d_k(v, t)$$
$$= \sum_{i+j+k=n} D_i(v, t)\beta d_j(w, t)\alpha d_k(w, t) + 2\sum_{i+j+k=n} D_i(w, t)\alpha d_j(v, t)\beta d_k(w, t) +$$
$$\sum_{i+j+k=n} D_i(w, t)\alpha d_j(w, t)\beta d_k(v, t) \qquad (7)$$

Furthermore,

$$D_n(w\alpha(w\beta v + v\beta w) + (w\beta v + v\beta w)\alpha w, t)$$
$$= D_n(w\alpha w\beta v + w\alpha v\beta w + w\beta v\alpha w + v\beta w\alpha w, t)$$
$$= D_n(w\alpha w\beta v, t) + D_n(w\alpha v\beta w, t) + D_n(w\beta v\alpha w, t) + D_n(v\beta w\alpha w, t)$$
$$= 2D_n(w\alpha v\beta w, t) + D_n(w\alpha w\beta v, t) + D_n(v\beta w\alpha w, t)$$
$$= 2D_n(w\alpha v\beta w, t) + \sum_{i+j+k=n} D_i(v, t)\,\beta d_j(w, t)\alpha d_k(w, t) +$$

$\sum_{i+j+k=n} D_i(w,t)\alpha\, d_j(w,t)\beta d_k(v,t)$ (8)

When we compare equations (7), (8) and $w\alpha v\beta w = w\beta v\alpha w$ we obtain

$2D_n(w\alpha v\beta w,t) - 2\sum_{i+j+k=n} D_i(w,t)\alpha d_j(v\ ,t)\beta d_k(w,t) = 0$

$2(D_n(w\alpha v\beta w,t) - \sum_{i+j+k=n} D_i(w,t)\alpha d_j(v\ ,t)\beta d_k(w,t)) = 0$

M is 2-torsion free Γ −ring, we obtain

$D_n(w\alpha v\beta w,t) - \sum_{i+j+k=n} D_i(w,t)\alpha d_j(v\ ,t)\beta d_k(w,t) = 0$

$D_n(w\alpha v\beta w,t) = \sum_{i+j+k=n} D_i(w,t)\alpha d_j(v,t)\beta d_k(w,t)$

Then D is Jordan generalized higher symmetric triple reverse bi-derivation on M.

Corollary 2.13

Every generalized higher symmetric reverse bi-derivation of 2 −torsion free Γ −ring M is Jordan generalized higher symmetric triple reverse bi-derivation on M.

Proof

Let $D = (D_i)_{i\in N}$ be generalized higher symmetric reverse bi-derivations on M.
This implies that $D = (D_i)_{i\in N}$ be Jordan generalized higher symmetric reverse bi-derivation on M.
By Proposition 2.12, we get $D = (D_i)_{i\in N}$ be Jordan generalized higher symmetric triple reverse bi-derivations on M

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