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RESEARCH ARTICLE - MATHEMATICS

Zero-Power Valued Mappings with Dependent Element of Associative Rings

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Abstract

Let R\` be a semi prime rings. An additive mapping L: R\`→R\` is called a zero-power valued on V over L is preserving V and if for each p belongs to V, that's meaning, there exists a positive integer n(p) > 1 such that L^(n(p))(p) = 0. The main purpose of this article is to study the notion of an additive mapping which is named zero-power valued mapping with dependent element of a semi prime ring. As a factual information, we deliver some results concerning that.

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Keywords: Free action, semi-prime ring, derivation, commuting mapping, skew-commuting mapping, dependent element, centralizer.

1. Introduction

The contemporary concept of an abstract ring was introduced in 1914 by Fraenkel in his work entitled "On zero divisors and the decomposition of rings". An important inquiry in the realm of Ring Theory pertains to the circumstances that result in the commutativity of a ring.The impetus for this study has been derived from the scholarly contributions of Mehsin Jabel [1]. and Mehsin Jabe and Dalal Ibraheem [2]. Numerous scholars have undertaken the examination of the concept of free action on operator algebras. The individuals in question are Murray and von Neumann [3] and von Neumann [4] introduced the notion of free action on a belian von Neumann algebras and used it for the construction of certain factors (see Dixmier [5]), Kallman [6]. There is a significant focus on the advancement of the preceding notion of unrestricted agency. The author extended the concept of unrestricted action of automorphisms on von Neumann algebras, which may not be commutative, by including the concept of dependent components inside an automorphism. Additionally, Chorda et al [7]. The idea of freely acting automorphisms in C*-algebras was extended. More information is available in [8] [9] [10] [7] [11] [12] [13] [14]

by incorporating dependent components connected with these automorphisms. Numerous scholars have shown a keen interest in examining dependent components within the framework of operator algebras.

In this work, we study and investigate the behavior of free action or (named act freely) mapping on associative ring $R`$, when this mapping acts as a zero-power valued on $R`$.

2. Preliminaries

In this context, the symbol "$R`$" will be used to denote an associative ring with the center denoted as Z($R`$). The commutator $[p, \omega]$ are supposed to be labeled as usual, where p and ω represent the elements involved in the commutation. The fundamental commutator identities, namely $[p\omega$, z]=$[p$,z]ω + $p[\omega$, z] and $[p$, ωz] = $[p, \omega]$z + $\omega[p$, z], will be used in our analysis. A ring R` is said to be n-torsion free, where n $\neq$ 0 is an integer, if for each element $p$ in $R$` such that np = 0, it follows that p= 0. It is important to note that a ring R` is a considered prime if the condition a$R$`b=(0) means that either a = 0 or b = 0. On the other hand, a ring R` is classified as semi-prime if the condition a$R$`a = (0) suggests that a = 0. In general, it may be said that a ring is considered prime if it is semi-prime, but it cannot be universally concluded that the reverse is true. A mapping d: R` $\rightarrow$ $R$` that satisfies the equation d$(p\omega)$ = d$(p)\omega$ + pd(ω) , $\forall\, p, \omega$ in R` is referred to as a derivation in mathematics. If we consider a real number a, then the mapping d: $R$` $\rightarrow$ R` defined as d$(p)$ =[a, $p$] may be seen as a derivation on $R$`. The mathematical concept referred to as "inner derivation" is applicable to the field of real numbers, denoted as R`. Let $\eta$ be an automorphism that maps a ring $R$`. A mapping d: R`$\rightarrow$ $R$` is referred to as a η- derivation if the equation d$(p\omega)$ = d$(p)\omega + \eta(p)$d(ω) is satisfied for any p, ω in the set of real numbers, denoted as R`. It should be noted for the mapping, denoted as d = $\eta-$ I, where I represents the identity mapping on the set of real numbers, is a η-derivation. Naturally, the notion of a η- derivation extends the notion of a derivation, since any I- derivation may be considered a derivation. η- derivations may be further generalized as (σ, η)-derivations. Furthermore, let σ and η are automorphism mappings of the ring R`. An additive mapping d: $R$`$\rightarrow$ R` is referred to as a $(\sigma, \eta) -$ derivation if the equation d$(p\omega)$ = d$(p)\sigma(\omega)$ + $\eta(p)$d$(\omega)$ holds for all elements $p$ and $\omega$ in $R$` . η-derivations and (σ, η)- derivations have been extensively used in several contexts. Specifically, under the context of solving certain functional equations. Moreover, in the context of ring theory, an additive mapping L from a ring R` to itself is referred to as a generalized derivation, accompanied along with the corresponding derivation d, if there is a situation a derivation d of $R$` where $L(p\omega)$ = $L(p)\omega$ + pd(ω) holds for all elements p and ω in R`. In general, the notion of a generalized derivation encompasses the two notions of a derivation and a left centralizer, where $L$ is equal to d and d is equal to 0, respectively (for more details, see to B. Hvala [15]). Furthermore, a mapping d: $R$` $\rightarrow$ R` is said to be centralizing (respectively, commuting) if [d$(p), p]$ belongs to the center of the ring $R$` (respectively, [d$(p), p]$ equals zero) for every p in R` . In this discussion, we present the definition of a generalized (1, $\eta$) -derivation in the context of a ring $R$`, where η is an automorphism mapping. A mapping L: R` $\rightarrow$ $R$` is considered a generalized (1, η)- derivation if it satisfies the condition $L(p\omega)$ = $pL(\omega)$ + d$(p)\eta(\omega)$ for all elements p and η in R`. Here, d is a (1, $\eta$)-derivation of $R$`.

Furthermore, a mapping L: R` $\rightarrow R$` is referred to as a left (or right) centralizer if it satisfies the condition $L(p\omega) = L(p)\omega$ (or $L(p\omega) = pL(\omega)$) for any p, ω in R`. Moreover, let $R$` be a ring. We define the left centralizer of R`, denoted as La$(p)$, as the set of element a$p$, where an is an elements of $R$` and p is an element of R`. Similarly, we define the right centralizer of $R$`, denoted as R`a$(p)$, as the set of elements $p$a, where $p$ is an element of $R$` and an is an element of R`. A mapping $L$ is said to be skew-centralizing on a subset U of $R$` (respectively, skew-commuting on a subset U of R`) if the condition $L(p)p$ + $pL(p)$ is satisfied $\forall\, p$ in U (respectively, the condition $L(p)p + pL(p)$ = 0 is satisfied $\forall\, p$ in U). According to the research conducted by Laradji and Thaheem [11], an element "a" in the set of real numbers ($R$`) is referred to as a dependent element of a mapping L: R`$\rightarrow$ $R$` over the equation $L(p)$a = ap holds true for every p in the set of real numbers (R`). A mapping $L$: $R$` $\rightarrow$ R` is referred to be a free action, or acting freely, on $R$` if the sole element over R` that is dependent on $L$ is zero. The absence of non-zero nilpotent dependent elements in a mapping $L$: $R$`$\rightarrow$ R` inside a semi-prime ring $R$` is shown more precisely in reference [11]. In the context of a mapping L: R` $\rightarrow$ $R$`, the notation D(F) represents the set of all elements in F that are dependent. If there are any ring theoretic ideas used in this context that have not been explicitly defined, we kindly direct the reader to the work of I. N. Herstein. [16].

Additionally , Let $\emptyset \neq$ V $\subseteq$ R`. Then a mapping L: $R$` $\rightarrow$ R` is called a zero-power valued on V over L is preserving V and if for each x belong to V. that's meaning , there exists a positive integer n$(p) > 1$ such that $L^{n(p)}(p)$ = 0.

3. The Results

In this section, we put the results of this article.

Theorem 3.1

Suppose $R`$ is a semi-prime ring, $\eta$ is automorphism mapping over $R`$ and $L: R` \to R`$ is a generalized (1, η)-derivation with associated (1, η) derivation d. If a is a dependent element over L and L is zero-power valued on $R`$. Then

(i) a ∈ D(p+d),

(ii) a = 0.

Proof:

(i) Assume a ∈ D(L). That means:-

$L(p)\mathrm{a} = \mathrm{a}p \quad . \forall\, p \in R`.$ (1)

Replacing p by $p\omega$, we find that

$pL(\omega)\mathrm{a} + \mathrm{d}(p)\eta(\omega) = \mathrm{a}p\omega \quad . \forall\, p,\ \omega \in R`.$ Then

$\omega\mathrm{a}\omega + \mathrm{d}(\omega)\eta(\omega)\mathrm{a} = L(p)\mathrm{a}\omega$. (2)

$\mathrm{d}(p)\eta(\omega)\mathrm{a} = 0. \quad \forall\, p, \omega \in R`$ (3)

Writing $\omega = \omega\kappa$, we see that

$\mathrm{d}(\omega)\eta(\omega)\eta(\kappa)\mathrm{a} = 0. \quad \forall\, p, \omega, \kappa \in R`.$ (4)

Right-multiplying (3) b$\omega\kappa$,we observe

$\mathrm{d}(p)\ \eta(\omega)\mathrm{a}\kappa = 0. \quad \forall\, p, \omega, \kappa \in R`.$ (5)

Subtracting (5) and (4), we obtain

$\mathrm{d}(p)\eta(\omega)(\eta(\kappa)\mathrm{a}\text{-}\mathrm{a}\kappa) = 0 \quad \forall\, p, \omega, \kappa \in R`.$

Due to η is automorphism mapping, then

$\mathrm{d}(p)L(\eta(\kappa)\mathrm{a}\text{-}\mathrm{a}\kappa) = 0. \ \forall\, p, \kappa \in R`.$

Putting $L = (\eta(\kappa)\mathrm{a} - \mathrm{a}\kappa)\ L\ \mathrm{d}(p)$ with using $R`$ is semi-prime ring.

We deduce that $\mathrm{d}(p)(\eta(\kappa)\mathrm{a} - \mathrm{a}\kappa) = 0$. This result yeilds

$\mathrm{d}(p)\eta(\kappa)\mathrm{a} = \mathrm{d}(p)\mathrm{a}\kappa \qquad \forall\, p,\omega, \kappa \in R`.$ (6)

Putting (6) in (2) yields the relation

$p\mathrm{a}\omega + \mathrm{d}(p)\mathrm{a}\omega = L(p)\mathrm{a}\omega.$

Obviously, this relation has the following fashion

$(p\mathrm{a} + \mathrm{d}(p))\mathrm{a}\omega = L(p)\mathrm{a}\omega\ ,\ \forall\, p, \omega \in R`.$ (7)

Since L is zero-power valued on $R`$, there exists an integer n($p$) > 1 such that $L^{n(p)-1}(p) = 0$, ∀ $p \in R`$.

Replacing p by $p - L(p) + L^2(p) +, \ldots ,+ (-1)^{n(p)-1}L^{n(p)-1}(p)$ in above relation ∀ $p \in R`$, we conclude that

$(p\mathrm{a} + \mathrm{d}(p))\mathrm{a}\omega = 0$. Due to $R`$ is semi-prime ring, we arrive to

$(p\mathrm{a} + \mathrm{d}(p))\mathrm{a} = 0$. Employing (8) in relation (7), we find that (8)

$L(p)\mathrm{a}\omega = 0$. Writing $\omega = \omega L(p)\mathrm{a}$ with using $R`$ is semi-prime ring, we deduce that

$L(p)\mathrm{a} = 0$. Consequently, relation (8) modifies to

$(p\mathrm{a} + \mathrm{d}(p))\mathrm{a} = L(p)\mathrm{a} = \mathrm{a}p$. In other words, we observe

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$(p$a + d$(p))$a = ap. According to relation (1), we find that

$((\omega$ + d$(p))$a-pa$)\omega$ = 0. Indicate to R\` acts as a semi-prime ring, we conclude that $(p$ + d$(p))$a = ap.

Consequently, we have the result a ∈ D(p+d) which is the requirement.

(iii) We put the result $L(p)$a = 0 in expression (1) with using R\` is semi-prime ring. It is enough to have the requirement result.

By applying the same manner with necessary variations, one can prove the following corollaries even without the characteristic assumption on L has a dependent element of L.

Corollary 3.2

Suppose R\` is a semi-prime ring, $\eta$ is automorphism mapping over $R$\` and L: R\` → $R$\` is a mapping. If L is (1, η)-derivation of semi-prime rings over L is zero-power valued on R\`, then a ∈ Z($R$\`) and L is free action on R\`.

Proof:

Suppose L = d, then d is a (1, η)-derivation.

Applying Theorem 3.1, we note that $(p + L(p))$a = ap.Then

pa + $L(p)$a = ax. Due to L is zero-power valued on R\`, there is an integer n$(p)$ > 1 such that $L^{n(p)-1}(p)$ = 0, ∀ $p \in R$\`.

Replacing p bω $p - L(p) + L^2(p)$ + ,…, + $(-1)^{n(p)-1}L^{n(p)-1}(p)$ in above relation, ∀ $p \in R$\`, we note that

pa = ap , ∀ $p \in R$\`. Clearly, a ∈ Z($R$\`) and L is free action on R\`.

Corollary 3.3

Assume R\` is a semi-prime ring, $\eta$ is automorphism mapping over $R$\` and L: R\` → $R$\` be a additive mapping satisfying $L(p\omega) = L(p)\eta(\omega)$ ∀ p, $\omega \in R$\` such that $L$ acts as a zero-power valued on $R$\`. If a ∈ D(L), then a ∈ Z(R\`) and $L$ is free action on $R$\`, where L is inner (1, η)-inner derivation.

Proof:

First of all, we take $pL(\omega)$ = 0. Applying the assumption in Theorem 3.1, we note that $L(p\omega)$ = d$(p)\eta(\omega)$ ∀ $p, \omega \in R$\` and a ∈ D($L$). Consequently, we see that $(L(p)\eta(\omega))$a = a$p\omega$ ∀ $p, \omega \in R$\`. Based on η is an automorphism mapping over R, we find that

$L(p)\omega$ a = a$p\omega$ ∀ $p, \omega \in R$\`. Depending on the fact that $L$ is zero-power valued on $R$\`, there exists an integer n(p) > 1 such that $L^{n(p)-1}(p)$ = 0, ∀ $p \in R$\`.

Replacing p bω $p - L(p) + L^2(p) + \ \ldots\ + (-1)^{n(p)-1}L^{n(p)-1}(p)$ in above relation, ∀ $p \in R$\`, we conclude that

a$p\omega$ = 0. ∀ p, $\omega \in R$\`. Writing $\omega$ = a and $p$ = $R$\`.

We demonstrate that aR\`a = 0.

Applying the fact that R\` has semiprimeness property, we find that

A = 0. Thus, we observe that a ∈ Z(R\`) and $L$ is free action over $R$\`.

Theorem 3.4

Assume R\` is a 2-torsion free semi-prime ring, $\eta$ is automorphism mapping over $R$\` and L: R\` → $R$\` be a generalized (1, η) derivation with associated (1, η) derivation d such that L acts as a zero-power valued on R\`. A mapping $\eta$: $R$\` → R\` defined by $\eta(p) = L(p)p + pL(p)$ ∀ $p \in R$\`. Then

(i) η is free action on R\`

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(ii) L is skew-commuting mapping of R`.

Proof:

(i) At beginning, we assume that a ∈ D(η), then

$\eta(p)$a = ap ∀ p ∈ R`.

($L(p)p$ + $pL(p)$) = ap, ∀ p ∈ R`. (9)

Replacing p by $p + \omega$ with using (9), we find that

($L(p)\omega$ + $L(\omega)p$ + $L\omega(p)$ + $pL(\omega)$)a = 0. ∀ p, ω ∈ R`. (10)

Writing ω by ωp , we observe that

($L(p)p\omega$ + $pL(\omega)p$ + d(p) $\eta(\omega)p$ + $p\omega L(p)$ + p^2 +$L(\omega)$ + pd(p)$\eta(\omega)$)a = 0. Due to the fact, that η is automorphism mapping over R`, then (($pL(\omega)p$ + $p\omega L(p)$ + $p^2L(\omega)$) + $L(p)p\omega$ + pd(p)ω + d(p)ωp)a = 0. Moreover, we arrive to

(p($L(\omega)p$ + $pL(p)$ + $pL(\omega)$) +$L(p)p\omega$ + pd(p)ω + d(p)ωp)a = 0.

According to relation (10), we find that

(-$pL(p)\omega$ + $L(p)p\omega$ + pd(p)ω + d(p)ωp)a = 0. ∀ p, ω ∈ R`.

(($L(p)p$ - $p(p)$)ω + pd(p)ω + d(p)ωp)a = 0. ∀ p, ω ∈ R`.

Replacing ω by a with employing (9), we conclude that

(ap -2$pL(p)$)a + pd(p)a + d(p)ap)a = 0. ∀ p ∈ R`. (11)

In relation (11), replacing p by –p over using the result with (11), we see that

2(-2$pL(p)$)a + pd(p)a + d(p)ap)a = 0. ∀ p ∈ R`.

Basically R` acts as a 2-torsion free, we observe that

(-2$pL(p)$)a + pd(p)a + d(p)ap)a = 0. ∀ p ∈ R`. (12)

Since L is zero-power valued on R, there exists an integer n(p) > 1 such that , $L^{n(p)-1}(p)$ = 0. ∀ p ∈ R`.

Replacing p by $p - L(p)$ + $L^2(p)$ + … + $(-1)^{n(p)-1}L^{n(p)-1}(p)$ in Expression (12), ∀ p ∈ R`, we deduce that

(pd(p)a + d(p)ap)a = 0. ∀ p ∈ R`.

Substituting this result in (11), we arrive to

(ap-2$pL(p)$)aa = 0. ∀ p ∈ R`. Consequently, we note that

apa = 0. ∀ p ∈ R`. Due to the fact that $R$` is semi-prime ring, we find that

a = 0. Hence, we arrive to η is free action on R`. This is the required result.

(ii) Putting the result of branch (i) which is a = 0 in relation (9), we satisfy L is skew-commuting mapping of R`.

Theorem 3. 5

Take R` as a semi-prime ring and a mapping $L$: $R$` → R` as a zero-power valued on $R$`. Then a ∈ D(d) if a ∈ Z(R`) and $L(p)$a = 0. ∀ $p$ ∈ $R$`.

Proof:

Given that a ∈ D(d), then

$L(p)$a = [a, p]a , ∀ p ∈ R`. (13)

Replacing p by $p\omega$ in (13), we conclude that

$L(p)\omega\mathrm{a} + pL(\omega)\mathrm{a} = p[\mathrm{a}, \omega]\mathrm{a} + [\mathrm{a}, p]\omega\mathrm{a}\ \forall\ p, \omega \in R`$. (14)

From (13) and (14), we observe that

$L(p)\omega\mathrm{a} = [\mathrm{a}\ ,p]\omega\mathrm{a}, \forall\ p, \omega \in R`$. (15)

Right-multiplying (15) by κ, we obtain

$L(p)\omega\mathrm{a}\kappa = [\mathrm{a}, p]\omega\mathrm{a}\kappa, \forall\ p, \omega, \kappa \in R`$. (16)

In expression (14), replacing ω by $\omega\kappa$ over subtracting this result with (16), we observer that

$L(p)\omega[\mathrm{a}, \kappa] = [\mathrm{a}, p]\omega[\mathrm{a}\ ,\kappa],\ \forall\ p, \omega,\kappa \in R`$. (17)

Left-multiplying (17) by p, based find that

$pL(p)\omega[\mathrm{a}, \kappa] = p[\mathrm{a}, p]\omega[\mathrm{a}\ ,\kappa],\ \ \forall\ p, \omega, \kappa \in R`$. (18)

Writing ω by $p\omega$ in (17), we note that

$L(p)p\omega[\mathrm{a}, \kappa] = [\mathrm{a}, p]p\omega[\mathrm{a}, \kappa], \forall\ p,\ \omega\ ,\kappa \in R`$. (19)

Using relations (18) and (19), we conclude that

$(L(p)p + pL(p))\omega[\mathrm{a}, \kappa] = ([\mathrm{a}, p]p + p[\mathrm{a}, p])\omega[\mathrm{a}, \kappa],\ \ \forall\ p, \omega, \kappa \in R`$. (20)

Since L is zero-power valued on $R`$, there exists an integer n($p$) > 1 such that $L^{n(p)-1}(p) = 0, \forall\ p \in R`$. Replacing p by $p - L(p) + L^2(p) + \ldots + (-1)^{n(p)-1}L^{n(p)-1}(p)$. Hence, from (20), based conclude that

$([\mathrm{a}, p]p + p[\mathrm{a}\ ,p])\omega[\mathrm{a}, p] = 0.\ \forall\ p, \omega \in R`$. (21)

Left-multiplying (21) by p, we find that

$([\mathrm{a}, p]p + p[\mathrm{a}, p])\omega[\mathrm{a}, p]p = 0.\ \forall\ p, \omega \in R`$. (22)

Replacing ω by $p\omega$ in expression (21), and employing the result with (22), we observer that $([\mathrm{a}\ ,p]p + p[\mathrm{a}\ ,p])\omega([\mathrm{a}\ ,p] + p[\mathrm{a},p\ ]) = 0.\ \forall\ p, \omega \in R`$. Basically $R`$ is a semi-prime ring. Then

$[\mathrm{a}, p]p + p[\mathrm{a}, p] = 0.\ \forall\ p \in R`$.

Thus inner derivation η: $R` \rightarrow R`$ defined by $\eta = [\mathrm{a}, p]$ is skew-commuting. Hence , $\eta(p) = 0$, which implies $[\mathrm{a}, p] = 0$.

Consequently, we obtain a $\in$ Z($R`$).

From expression (13), we note that $L(p)\mathrm{a} = 0$.

Conversely, let a $\in$ Z($R`$) and $L(p)\mathrm{a} = 0$.

Then $L(p)\mathrm{a} = [\mathrm{a}, p]\mathrm{a} = 0$, which implies a $\in$ D(d).

By this step we complete the proof.

Theorem 3.6

Assume $R`$ is a prime ring $\eta$: $R` \rightarrow R`$ is a generalized derivation over $R`$ and a $\in R`$ is an element dependent on $\eta$ = d + $L$ such that $L$ acts as a zero-power valued on $R`$, then either a $\in$ Z($R`$) or $\eta$ is identity mapping of $R`$.

Proof:

We have the relation $\eta(L)\mathrm{a} = \mathrm{a}p, \forall\ p \in R`$. (23)

Replacing p by $p\omega$ in (23), we conclude that

$(\eta(p)p + p\mathrm{d}(\omega))\mathrm{a} = \mathrm{a}p\omega, \forall\ p, \omega \in R`$. (24)

Considering that η can be expressed in the form η = d + L, wherein L is a left centralizer.

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Now, replacing d(ω)a by $\eta(\omega)$a - $L(\omega)$a in relation (24),

with applying expression (23), yields the result

$\omega(p)\omega$a + [p, a] - $\omega p L(\omega)$a = 0. $\forall\ p, \omega \in R$` . (25)

Writing ω by $\omega\eta(p)$ in (25), we observe

$\eta(p)\omega\eta(p)$a + [p, a]$\omega\eta(p)$ - $pL(\omega\eta(p))$ a = 0 . $\forall\ p, \omega \in R$`. (26)

Investment the fact that L is left centralizer, then (26), becomes

$\eta(p)\omega\eta(p)$a + [p, a]$\omega\eta(p)$ - $pL(\omega)\eta(p)$a = 0. (27)

Using relation (23) in (27), we harvest the relation

$\eta(p)\omega$ap +[p, a]$\omega\eta(p)$ - $pL(\omega)$ap = 0. $\forall\ p, \omega \in R$`. (28)

Right –multiplying expression (25), by p implies to

$\eta(p)\omega$ap + [p, a]ωp - $pL(\omega)$ap = 0. $\forall\ p \in R$` . (29)

Indicate to p acts as a zero-power valued on R , there exists an integer n(ω) > 1 such that $h^{n(\omega)}(\omega)$ = 0, $\forall\ \omega \in R$`.

Replacing ω by ω − h(ω) + $h^2(\omega)$ + … + $(-1)^{n(\omega)-1}h^{n(\omega)-1}(\omega)$ in relations (28) over (29), respectively, we harvest the relations

$\eta(p)\omega$ap + [p, a]$\omega\eta(p)$ = 0. $\forall\ p, \omega \in R$`. (30)

$\eta(p)\omega$ap + [p, a]ωp = 0. $\forall\ p \in R$` . (31)

Subtracting (30) and (31), we find that

[p, a]$\omega(\eta(p)$ - $p)$ = 0. $\forall\ p \in R$`.

Then $[p, a]R`\left(\eta(p) - (p)\right) = 0$. Since $R$` is prime ring , based either get [p, a] = 0. $\forall\ p \in R$`, which yields to a ∈ Z($R$`) or $\eta(p) = p, \forall\ p \in R$`.

Theorem 3.7

Suppose R is a prime ring and η: R` → $R$` is a non-zero (σ, η)-derivation such that σ acts as a zero-power valued on R`, then a ∈ Z($R$`), where a ∈ R`.

Proof:

We have the relation $\eta(p)$ a = ap, $\forall\ p \in R$`. (32)

Replacing p by $p\omega$,we obtain $\eta(p)$σ (ω)a + $\eta(r)\eta(\omega)$a = a$p\omega$, $\forall\ p, \omega \in R$`. As stated by (32), one can replace $\eta(\omega)$a by aω above relationship,

That provides $\eta(p)$σ (ω)a + ($\eta(p)$a - a$p)\omega$ = 0. $\forall\ p, \omega \in R$`. (33)

Replacing ω by $\omega\kappa$ in (33), we obtain

$\eta(p)$σ (ω) σ(κ)a + $\eta(p)$a - a$p)\omega\kappa$ = 0. $\forall\ p, \omega, \kappa \in R$`. (34)

Right- multiplying (33) by κ, we find that

$\eta(p)$σ ω)(aκ) + ($\eta(p)$a - a$p)\omega$z = 0. $\forall\ p, \omega, \kappa \in R$`. (35)

Subtracting (35) from (34), we get $\eta(p)$σ(ω)(σ(κ) a - aκ) = 0. $\forall\ p, \omega, \kappa \in R$`. Stated otherwise, based have

$\eta(p)\omega$(σ (κ)a - aκ)=0. $\forall\ p, \omega, \kappa \in R$`.

Then $\eta(p)R$`(σ(κ)a – aκ) = 0.

Since R` is prime and η is non-zero (σ, η)-derivation, we arrive to

$\sigma(\kappa)a = a\kappa.\ \forall\ \kappa \in R`.$ (36)

Since σ is automorphism mapping of $R`$. Then, in additional terms from (36), we have $\kappa a = a\kappa$ $\forall\ \kappa \in R`$.

Due to η is automorphism mapping of $R`$ from (33), we obtain

$\eta(p)\ \sigma(\omega)a + (pa - ap) = 0.\ \forall\ p, \omega \in R`.$

Based on σ is zero-power valued on $R`$, there exists an integer $n(p) > 1$ such that $\sigma^{n(\omega)}(\omega) = 0$, $\forall\ \omega \in R`$.

Replacing ω by $\omega - \sigma(\omega) + \sigma^2(\omega) + \ldots + (-1)^{n(\omega)-1}\sigma^{n(\omega)-1}(\omega)$, we harvest $pa - ap = 0.\ \forall\ p \in R`$, which leads to $a \in Z(R`)$.

This completes the proof of the theorem.

Conclusion:

Let R` be a ring. An additive mapping L: R` → R` is called a derivation if $L(p\omega) = L(p)\omega + pL(\omega)$ holds for all p, $\omega$ ∈ R`. The study of derivations in prime rings was initiated by E.C. Posner in 1957. Over the last several years, a number of authors have studied commutativity theorems for prime rings and semi-prime rings, admitting automorphisms or derivations on appropriate subsets of R`. Let $\phi \neq U \subseteq R`$. Then a mapping L: R` → R` is called a zero-power valued on U if L is preserving U and if for each p belongs to U. That's meaning, there exists a positive integer n(p) > 1 such that $L^{n(p)}(p) = 0$. An element a ∈ R` is called a dependent element of a mapping L: R` → R` if L(p)a = ap holds for all p ∈ R`. In fact, a mapping L: R` → R` is called a free action or act freely on R` if zero is the only dependent element of L.

The goal of this paper is to study the behavior of zero-power valued mapping of associative rings satisfying some algebraic conditions and collect information about the free action and skew-commutative structure of these rings.

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