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RESEARCH ARTICLE - Physics

Study of changing eight entangled states of four qubits using quantum gates

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Abstract

The study aims to use a quantum gate to create eight entangled states in a quantum circuit consisting of four qubits. Where using gates $|q_0q_1q_2q_3\rangle$ entanglement states can be shown, by using quantum gates that give the same arrangement as the circuit gates and for the same inputs, taking into account the outputs of a quantum circuit containing entangled functions. The study explains how to use control gates to isolate specific qubits. And measuring their condition, which allows for studying the interconnection between them. Study methodology: Using The results indicated that the change in the output states occurred effectively in all eight state (The quantum circuit was simulated using the *MATLAB R2023b* quantum computing language) and that similar results could be obtained using gates operating on three qubits, but the entangled state created by the CNOT gates must be preserved. The results also indicated that it is not possible to create 8 states for one state. The output of a two-qubit quantum circuit $|q_0q_1\rangle$, but this can be generated for a 3-qubit quantum circuit, and we also need three Hadamard gates and this applies to $((2+n)\_qubits)$, where $n \geq 3$ It represents the number of qubits in the circuit.

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Keywords: Qubit state , Quantum entanglement, Quantum superposition , Quantum gates

1. Introduction

Quantum computing is a rapidly developing field, especially following the advancements in quantum mechanics [1]. This field relies on quantum mechanics to enable significant computational capabilities that surpass those of classical computing in solving complex problems [2]. A core principle of quantum mechanics is quantum superposition, which suggests that a physical entity in a quantum system can exist in two states simultaneously before measurement [3]. In the context of quantum computing, this principle is reflected by considering the outputs of a quantum circuit as the states of its qubits, with each qubit capable of transmitting a blend of information concurrently [4]. Within any Hilbert space, a quantum state is represented as a vector in $(\boldsymbol{R^2})^{\otimes \boldsymbol{n}}$ [5], which allows for the description of a bit string on a computational basis as a superposition of multiple qubits states [6]. The general equation for a single qubit, representing a vector in two-dimensional Hilbert space, confirms that a qubit itself is such a vector [7]. The role that quantum computing plays is fundamental and vital, as quantum entanglement states are the basis of complexity in complex mathematical operations, and if entanglement is identified, many complex computational problems will be solved. In this study, which aims to aims to use a quantum gate to create eight entangled states in a quantum circuit consisting of four qubits. Where using gates $|\boldsymbol{q_0q_1q_2q_3}\rangle$ entanglement states can be shown, by using quantum gates that give the same arrangement as the circuit gates and for the same inputs, We will discuss some concepts and theories related to quantum computing, as The importance of this study is due to the fact that it is a simple and effective approach to creating quantum entanglement states in quad-qubit quantum circuits, which is considered of great importance in the field of quantum computing for various reasons, including: facilitating the construction of complex quantum circuits, and it also helps to develop the understanding of quantum entanglement states in quantum circuits. It also It works to improve the performance of quantum devices and reduce the cost of building them, which is considered expensive. It also contributes to the diversity of applications that depend on quantum devices, which supports and contributes to achieving sustainability for these applications. A quantum computing-based framework could be developed to support numerical optimization and multivariate distribution analysis, helping to design more efficient and effective quantum neural networks[8,9].

2. Basic and theoretical concepts

2.1 . Basic concepts

There are some important concepts and principles that must be noted, including :

- Quantum entanglement: Quantum entanglement can be defined as a phenomenon that uniquely connects multiple qubits. The state of one qubit cannot be described independently of the states of other qubits. In this case, the state of one qubit can be measured to reveal information about the states of other entangled qubits without having to measure them directly.
- Quantum gates: They are tools used to control qubits in a quantum circuit, and are used to build complex quantum circuits. These gates enable the implementation of logical operations on qubits, such as NOT, CNOT, and Hadamard. theoretical approach and basic concepts

2.2. theoretical concepts

The general equation for a single qubit, which represents a vector in a two-dimensional Hilbert space, is given by :

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (1)$$

Where: α and β are complex coefficients representing vector components in the basis of $\{|0\rangle, |1\rangle\}$.

$|\alpha|^2$ represents the probability of measuring a qubit in the state $|0\rangle$.

$|\beta|^2$ represents the probability of measuring a qubit in the state $|1\rangle$.

The norm of a qubit equals 1, This is probability in quantum mechanics. Which means $\langle\psi|\psi\rangle = 1$, $|\alpha|^2 + |\beta|^2 = 1$. So a qubit is a quantum state vector (a function of the binary degree of freedom $\{0,1\}$) which represents a superposition of the states of a single qubit [10].the state qubit $|\psi\rangle$ It consists of two states $\{|0\rangle, |1\rangle\}$.the computational basis for them in$(\mathbb{R}^2)^{\otimes n}$equal $\mathbb{R}^2$ for $n = 1$ for single qubit [11] ;

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}, |1\rangle = \begin{bmatrix}0\\1\end{bmatrix} \quad (2)$$

The system that is formed with n-qubits needs the property of the interaction of qubit states, and this property is "quantum entanglement", which when measuring the system gives useful results in quantum information science. The states are entangled for an output state that must be subject to a special arrangement that depends on the value of the input state of the circuit and on the type of quantum gates that apply in it [12]. For example to show the entanglement states of two qubits :

$$|\psi_{AB}\rangle = |\psi_A\rangle \otimes |\psi_B\rangle \quad (3)$$

The entangled state vector $|\psi_{AB}\rangle$, To prove that they are entangled, we must work on the density of the matrix through evaluation; $\rho_{AB} = |\psi_{AB}\rangle\langle\psi_{AB}|$ [10]. That is, the entangled state must be inseparable; $|\psi_{AB}\rangle = |\psi_A\rangle_1 \otimes |\psi_B\rangle_1 + |\psi_A\rangle_2 \otimes |\psi_B\rangle_2$. Quantum circuits consist of n-qubits whose outputs are subject to unitary transformations by quantum gates. Depending on the type of inputs, outputs are produced whose states of the circuit may be separable states [13]. There are many circuits whose outputs are entangled states, such as Bell states for one qubit, which represent the entanglement of two-qubit states and are also generalized to a circuit containing more than two qubits, whose outputs are GHZ states [12].There is more than one way to produce entangled states, We will deal with a quantum circuit consisting of four qubits, and according to the quantum computational basis and connecting it to the basis vectors, we will obtain 16 states, Which we obtained through 2^n, where n is the number of qubits in the quantum circuit [14]. We worked for a state of $|0\rangle$ for each of the four qubits, and this shows us that the input state of the quantum circuit is : $\{ \ |q_0\rangle = |0\rangle, |q_1\rangle = |0\rangle, |q_2\rangle = |0\rangle, |q_3\rangle = |0\rangle \ \}$

input state for 4-qubit ; $|\psi_i\rangle = |q_0\rangle \otimes |q_1\rangle \otimes |q_2\rangle \otimes |q_3\rangle = |0000\rangle$

unitary transformations are the reason for realizing logic gates in quantum computing [15]. The quantum gate's operation is determined on the computational basis of the given spaces In quantum computing, this basis is represented by matrices ($2^n \times 2^n$) [16]. As a simple definition, the properties of a single qubit for unitary operations $(\hat{U})$ are represented by a matrix (2×2) [17], that is, the state of the single qubit is transformed into a new state by applying the gate ;

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad , \quad |\psi`\rangle = \alpha`|0\rangle + \beta`|1\rangle \quad , \ \alpha` = U_{11}\,\alpha + U_{12}\,\beta \quad , \ \beta` = U_{21}\,\alpha + U_{22}\,\beta$$

It can be expressed in the form of a matrix represented [18] ;

$$\begin{bmatrix} \alpha` \\ \beta` \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \rightarrow |\psi`\rangle \equiv U\,|\psi\rangle \quad ,\langle \psi` | \psi` \rangle = \langle \psi | \mathrm{U}^{\dagger} U \,|\psi\rangle$$

where :

- **i.** the identity matrix : $U^{\dagger}U = I \ \rightarrow \ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ \Longrightarrow \ \mathrm{I-gate}$
- **ii.** α and β are complex coefficients that represent the probabilities of the qubit being in the $|0\rangle$ and $|1\rangle$ states, respectively. They are probability values.
- **iii.** $U_{11}, U_{12}, U_{21}, U_{22}$: are the elements of the U_{gate} matrix.

And the corrected representation (ketbra) of the I-gate should be [19] :

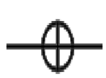
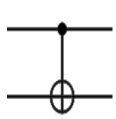
$$I = |0\rangle\langle 0| + |1\rangle\langle 1| \tag{4}$$

Dirac notation for the Outer product of the I-gate :

$$|0\rangle\langle 0| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad |1\rangle\langle 1| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \rightarrow \quad |0\rangle\langle 1| = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \ , \ |1\rangle\langle 0| = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Through this definition, we understand that quantum gates handle qubit states, which are represented as a state vector on the computational basis of states [20]. The following table shows the gates used that were applied to the quantum circuit, which do not create superposition (i.e. do not make the state of the qubit add $|0\rangle$ and $|1\rangle$ in the presence of amplitude and probability% that affect the circuit's outputs). Rather, these gates work to change the values of the qubits in the state, which consists of four qubits $|q_0q_1q_2q_3\rangle$, It becomes a new state. Through the table, we can understand which gate operates on one-qubit $|q_i\rangle$ and which gate operates on two-qubit $|q_iq_i\rangle$. Note that all gates can operate on four qubits $|q_iq_iq_iq_i\rangle$ and by inserting a tensor product$\{\otimes\}$ with the I-gate.

Table 1. Quantum gates used with an indication of their output and input [21].

Gate	Symbol	Matrix representation	Dirac representation	INPUT	OUTPUT
X (qubit flip)		$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\|1\rangle\langle 0\| + \|0\rangle\langle 1\|$	$\|0\rangle$ $\|1\rangle$ $\frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$ $\frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)$	$\|1\rangle$ $\|0\rangle$ $\frac{1}{\sqrt{2}}(\|1\rangle + \|0\rangle)$ $\frac{1}{\sqrt{2}}(\|1\rangle - \|0\rangle)$
CNOT (entan-gled states)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	$\|0\rangle\langle 0\| \otimes \mathbb{I}$ $+ \|1\rangle\langle 1\| \otimes \mathrm{X}$	$\|00\rangle$ $\|01\rangle$ $\|10\rangle$ $\|11\rangle$	$\|00\rangle$ $\|01\rangle$ $\|11\rangle$ $\|10\rangle$

SWAP (Swap the target qubits)		$\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}$	$\|00\rangle\langle 00\|$ $+\|11\rangle\langle 11\|$ + $\|01\rangle\langle 10\|+\|10\rangle\langle 01\|$	$\|00\rangle$ $\|01\rangle$ $\|10\rangle$ $\|11\rangle$	$\|00\rangle$ $\|10\rangle$ $\|01\rangle$ $\|10\rangle$

The gate responsible for the superposition in the quantum circuit and plays a major role in entanglement of states is the Hadamard gate, which gives a superposition of the state of the target qubit with an amplitude and probability% [21]. The matrix representation of the H gate :

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \tag{5}$$

the H gate follows the relationship $H^n = (2^n)$, $\boldsymbol{n}$; is the number of H gates and Where , $\boldsymbol{2^n}$; represents the number of states in an output state. For the same $\boldsymbol{n}$, the amplitude takes the value $1/\sqrt{2^n}$ .The Dirac representation of the H- gate ; $\{\ H = |+\rangle\langle 0| + |-\rangle\langle 0|\ \}$

$$H - gate|0\rangle = \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) = |+\rangle\ , H - gate|1\rangle = \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) = |-\rangle$$

Since we will be working to show 8 entangled states for the output of a quantum circuit consisting of 4-qubits, we will need three Hadamard gates.

$$\cdot\ H^1 - gate\ |q_0q_1q_2q_3\rangle = \frac{1}{\sqrt{2}}\ (\ |q_0q_1q_2q_3\rangle_1 + |q_0q_1q_2q_3\rangle_2\ )$$

$$\cdot\ H^2 - gate\ |q_0q_1q_2q_3\rangle = \frac{1}{2}\ (\ |q_0q_1q_2q_3\rangle_1 + |q_0q_1q_2q_3\rangle_2 + |q_0q_1q_2q_3\rangle_3 + |q_0q_1q_2q_3\rangle_4\ )$$

$$\cdot\ H^3 - gate\ |q_0q_1q_2q_3\rangle = \frac{1}{2\sqrt{2}}\ (\ |q_0q_1q_2q_3\rangle_1 + |q_0q_1q_2q_3\rangle_2 + |q_0q_1q_2q_3\rangle_3 + |q_0q_1q_2q_3\rangle_4$$
$$|q_0q_1q_2q_3\rangle_5 + |q_0q_1q_2q_3\rangle_6 + |q_0q_1q_2q_3\rangle_7 + |q_0q_1q_2q_3\rangle_8\ ) \tag{6}$$

2.2.1. A quantum circuit outputs eight entangled states

The negative sign of the amplitude of a state indicates that the phase angle is π. We found more than one arrangement to create eight entangled states from a circuit with the same gates and the same input $|0000\rangle$. Figure 1. shows the difference in the arrangement of the gates on the four qubits.

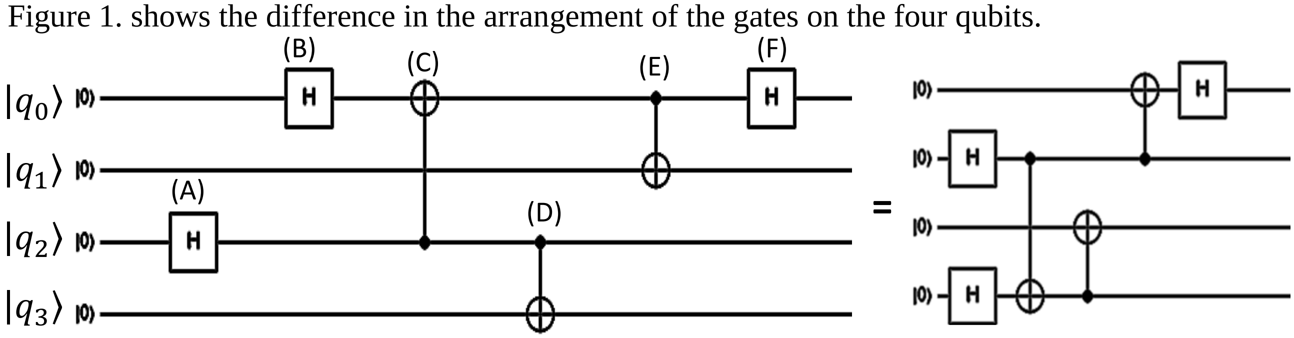


Fig. 1. A quantum circuit consisting of 4-qubits, whose outputs are entangled states.

the input $|q_0q_1q_2q_3\rangle = |0000\rangle$, Circuit details are as follows :

(A) The Hadamard gate acts on the third qubit $|q_2\rangle = |0\rangle$ and creates a state output consisting of two states .

$$H^1 - gate|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)$$

Taking the tensor product between the state above and the qubit $|q_3\rangle = |0\rangle$:

$$|\psi\rangle = |0\rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) = \left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|01\rangle\right)$$

Taking the tensor product between the state above and the qubit $|q_0\rangle = |0\rangle$ and $|q_1\rangle = |0\rangle$:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle) \otimes |0\rangle \otimes |0\rangle = \left(\frac{1}{\sqrt{2}}|0000\rangle + \frac{1}{\sqrt{2}}|0100\rangle\right)$$

The output state $|\psi\rangle$, It consists of two states, each state has a probability of 50% .

(B) The H gate acts on the first qubit $|q_0\rangle$, so the output state consists of four states with a probability of 25% for each state.

$$H^2 - gate|\psi\rangle = \left(\frac{1}{\sqrt{2}}|000\rangle \otimes \mathrm{H}|0\rangle + \frac{1}{\sqrt{2}}|010\rangle \otimes \mathrm{H}|0\rangle\right)$$

$$|\psi\rangle = \left(\frac{1}{\sqrt{2}}|000\rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) + \frac{1}{\sqrt{2}}|010\rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)\right)$$

The output state :

$$|\psi\rangle = \left(\frac{1}{2}|0000\rangle + \frac{1}{2}|0001\rangle + \frac{1}{2}|0100\rangle + \frac{1}{2}|0101\rangle\right)$$

(C) CNOT-gate ; control of the third qubit $|q_2\rangle$ and the target of the first qubit $|q_0\rangle$, When the CNOT gate is mediated by a qubit, meaning that the CNOT gate can operate on more than two qubits, the Dirac representation becomes : $CNOT = |0\rangle\langle 0| \otimes \mathbb{I} \otimes \mathbb{I} + |1\rangle\langle 1| \otimes \mathbb{I} \otimes \mathrm{X}$
It does not change the output state, but it will help us maintain entanglement when changing the location of the target H gate of the first qubit $|q_0\rangle$.

(D) CNOT gate; control of $|q_2\rangle$, target of $|q_3\rangle$, It will affect $|0100\rangle$ and $|0101\rangle$.

NOT: In Qiskit's convention, According to the indicators of the highest qubit, which is the most important (little endian convention) [22]. We will introduce the controlled gate, which is the X-gate.

It will become the Dirac representation of the CNOT gate ; $CNOT = |0\rangle\langle 0| \otimes \mathbb{I} + |1\rangle\langle 1| \otimes \mathrm{X}$

the CNOT gate acts on the output state (B) ;

$$CNOT - gate|\psi\rangle = \left(\frac{1}{\sqrt{2}} CNOT|00\rangle \otimes |00\rangle + \frac{1}{\sqrt{2}} CNOT|00\rangle \otimes |01\rangle + \frac{1}{\sqrt{2}} CNOT|01\rangle \otimes |00\rangle + \frac{1}{\sqrt{2}} CNOT|01\rangle \otimes |01\rangle\right)$$

for state $|0100\rangle$: $CNOT - gate\,|01\rangle \otimes |00\rangle$

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \otimes \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} + \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} \otimes \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix} |01\rangle \otimes |00\rangle$$

$$\begin{bmatrix}1\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} & 0\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\\0\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} & 1\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\end{bmatrix} + \begin{bmatrix}0\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix} & 1\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}\\1\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix} & 0\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}\end{bmatrix} |01\rangle \otimes |00\rangle = CNOT - gate|01\rangle \otimes |00\rangle$$

Performing a matrix representation calculation ;

$$CNOT-gate|01\rangle \otimes |00\rangle = \left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right) \otimes \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$CNOT-gate|01\rangle \otimes |00\rangle = |11\rangle \otimes |00\rangle = |1100\rangle$$

In the same way calculate the CNOT-gate $|0101\rangle$, We get ; $CNOT-gate|0101\rangle = |1101\rangle$

The output state :

$$|\psi\rangle = \left(\frac{1}{2}|0000\rangle + \frac{1}{2}|0001\rangle + \frac{1}{2}|1100\rangle + \frac{1}{2}|1101\rangle\right)$$

(E) CNOT gate, control of $|q_0\rangle$, target of $|q_1\rangle$, In the same way as the previous point, the gate will affect $|0001\rangle$ and $|1101\rangle$,we get an output state ;

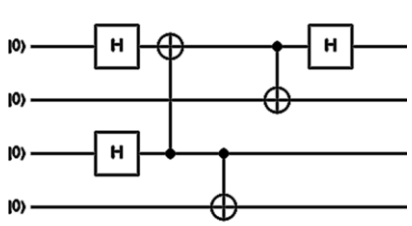
$$|\psi\rangle = \left(\frac{1}{2}|0000\rangle + \frac{1}{2}|0011\rangle + \frac{1}{2}|1100\rangle + \frac{1}{2}|1111\rangle\right)$$

(F) The last H^3 gate targets the first qubit $|q_0\rangle$, and gives an output state consisting of 8 entangled states like equation (6), with an amplitude equal to $\frac{1}{2\sqrt{2}}$ and with phase angles ranging $\{0, \pi, 2\pi\}$ and the probability of 12.5% for each state . If it had targeted the qubit $|q_1\rangle$, it would have given the same results as the states and the same amplitude, but some states differ in terms of the phase angle. The final output state ;

$$|\psi\rangle = \left(\frac{1}{2\sqrt{2}}|0000\rangle_1 + \frac{1}{2\sqrt{2}}|0001\rangle_2 + \frac{1}{2\sqrt{2}}|0010\rangle_3 - \frac{1}{2\sqrt{2}}|0011\rangle_4 + \frac{1}{2\sqrt{2}}|1100\rangle_5 + \frac{1}{2\sqrt{2}}|1101\rangle_6 + \frac{1}{2\sqrt{2}}|1110\rangle_7 - \frac{1}{2\sqrt{2}}|1111\rangle_8\right) \quad (7)$$

3. Results

We will work on changing the targeting of the gates to the qubits, and to keep the entangled states, the CNOT gates will be fixed while using the X gate and the SWAP gate, using the same calculation method in the previous section, Regardless of the phase angles, we can apply a methodology to show the eight entangled states. we can represent $|\psi\rangle..(7)$ in the computational basis state diagram of 4-qubits by following ; will notice the emergence of new states when changing the targeting of the H gate.



$\Longrightarrow |\psi\rangle =$

$|0000\rangle$	$|0001\rangle$	$|0010\rangle$	$|0011\rangle$
$|0100\rangle$	$|0101\rangle$	$|0110\rangle$	$|0111\rangle$
$|1000\rangle$	$|1001\rangle$	$|1010\rangle$	$|1011\rangle$
$|1100\rangle$	$|1101\rangle$	$|1110\rangle$	$|1111\rangle$

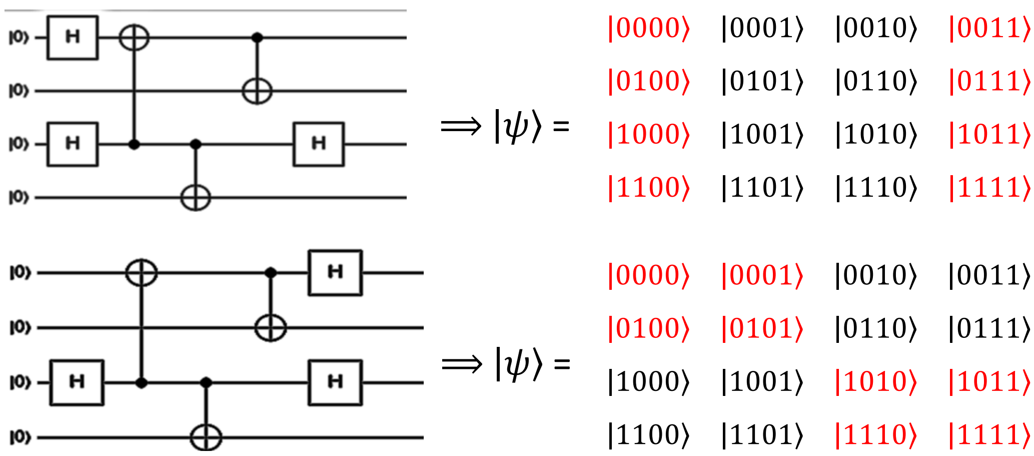


Fig. 2. Possibility of showing the eight entangled states using the H gate.

When using the X gate, new states will appear that did not appear in Fig. 2. Considering that its action is specific to one state (flip-gate ; It completely flips the state). the output state changes its state completely when using the X gate.

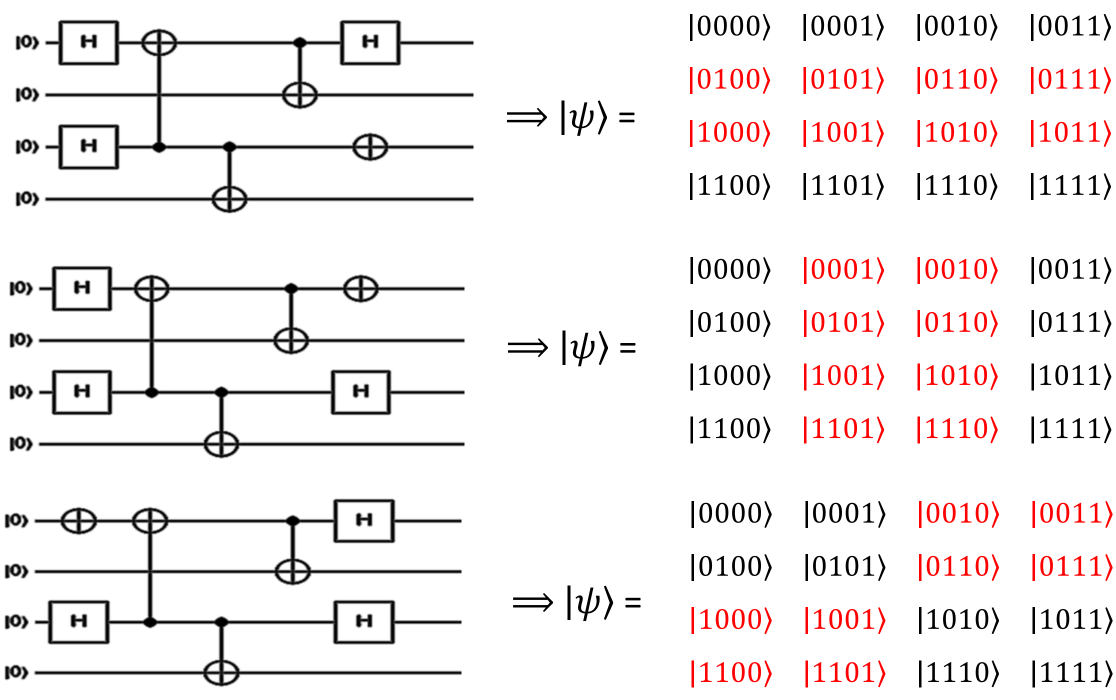


Fig. 3. Possibility of showing the eight entangled states using the H gate and X gate.

When using the SWAP-gate, we will exploit the permutation property of the qubit states in the state $|q_0q_1q_2q_3\rangle$, to show the outputs of divergent entangled states in the computational basis of the 4-qubit states, As in the following figure.

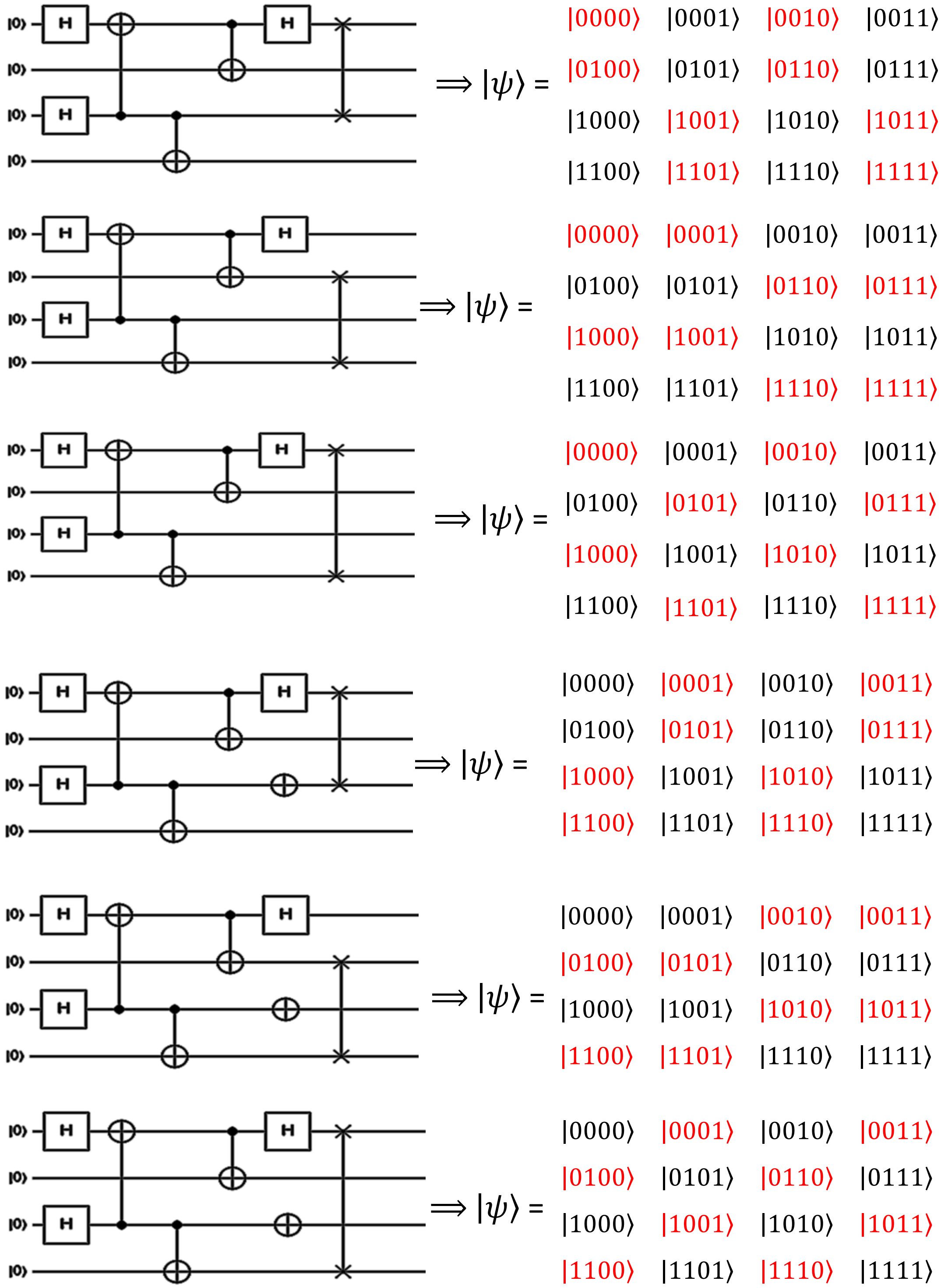


Fig. 4. shows the possibility of 8 entangled states appearing when using a SWAP gate on a quantum circuit.

As programming for some quantum circuits in this research, we used code in ***MATLAB R2023b*** [23], which gave results (outputs) for 4-qubits, represented by 8 states, with an amplitude of 0.35355. This is done by changing the targeting of the gate in the quantum circuit, or by adding a gate to the quantum circuit that matches the existing gates that take into account the values of the four qubits $|q_0q_1q_2q_3\rangle$.

```
disp('A)Figure 1.')
c = quantumCircuit(4);
s = simulate(c);
str = formula(s)
```

```
str = "1 * |0000>"
```

```
inState =
quantum.gate.QuantumState([1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0]')
```

```
inState =
  QuantumState with properties:
    BasisStates: [16×1 string]
     Amplitudes: [16×1 double]
      NumQubits: 4
```

```
s = simulate(c,inState);
str = formula(s,Basis="Z")
gates1=[hGate(3),hGate(1),cxGate(3,
1),cxGate(1,2),cxGate(3,4),hGate(1)
]
```

```
gates1 = 1×6 SimpleGate array with
gates:
            Id    Gate    Control    Target
             1     h                   3
             2     h                   1
             3     cx        3         1
             4     cx        1         2
             5     cx        3         4
             6     h                   1
```

```
c1 = quantumCircuit(gates1,4);
figure ; plot(c1)
disp('output state')
disp('B)Figure.2')
gates2=[hGate(3),hGate(1),cxGate(3,
1),cxGate(1,2),cxGate(3,4),hGate(3)
]
```

```
gates2 = 1×6 SimpleGate array with
gates:
            Id    Gate    Control    Target
             1     h                   3
             2     h                   1
             3     cx        3         1
             4     cx        1         2
             5     cx        3         4
             6     h                   3
```

```
c2 = quantumCircuit(gates2,4);
figure plot(c2) disp('output state')
s2 = simulate(c1,inState);
s1 = simulate(c2,inState);
ouStateA = formula(s1,Basis="auto")
ouStateB = formula(s2,Basis="auto")
disp('D)Figure 3')
gates3=[hGate(3),hGate(1),cxGate(3,1),cx
Gate(1,2),cxGate(3,4),hGate(1),xGate(3)]
```

```
gates3 = 1×7 SimpleGate array with
gates:
            Id    Gate    Control    Target
             1     h                   3
             2     h                   1
             3     cx        3         1
             4     cx        1         2
             5     cx        3         4
             6     h                   1
             7     x                   3
```

```
c3 = quantumCircuit(gates3,4);
figure plot(c3)
disp('output state')
disp('c)Figure 3.')
gates4=[hGate(3),hGate(1),cxGate(3,1),cx
Gate(1,2),cxGate(3,4),hGate(3),xGate(1)]
```

```
gates4 = 1×7 SimpleGate array with
gates:
            Id    Gate    Control    Target
             1     h                   3
             2     h                   1
             3     cx        3         1
             4     cx        1         2
             5     cx        3         4
             6     h                   3
             7     x                   1
```

```
c4 = quantumCircuit(gates4,4);
figure plot(c4)
disp('output state')
s4 = simulate(c3,inState);
s3 = simulate(c4,inState);
ouStateC = formula(s3,Basis="auto")
ouStateD = formula(s4,Basis="auto")
```

Fig. 5. The quantum circuit simulation code used: It shows the change of the 8 entangled states only using quantum gates. Note only the gates part.

***NOTE*:** to implement the SWAP gateway in Figure 4. the simulated code is the same. We only add to `gates` the gate code `swapGate( , ),` taking into account the gate parameters and the targeting of the qubits$|q_i q_i\rangle$.

***NOTE*:** all the outputs we obtained, which consisted of 8 entangled states, were of one input type, which is state$|0000\rangle$.

4. discussion

According to **Fig. 2.** eight entangled states can appear using an H gate on a single qubit. The H gate is a basic quantum gate used to create entangled states and to convert a qubit from a $|0\rangle$ state to a $|1\rangle$ state and vice versa. There are four states. Possible for input: $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$.

where :

- (0000): represents the entrance state of the qubit, which is $|00\rangle$, while (0001), (0010), (0011) are the other three possible entrance states of the qubit.
- $|11\rangle$: represents the entrance state of the qubit, while (1100), (1101), (1110), (1111) are the other three states.

Following the application of the H-gate to a single qubit, **Fig. 3.** illustrates the H-gate being applied to each qubit; thus, 64 possible states are possible, with each qubit occupying one of eight possible states. Given that there are 64 possible states and each qubit has eight states, the output has 64 possible states. As illustrated in Figure No. 3.

Fig. 4. illustrates the potential for eight entangled states to manifest in a quantum circuit comprised of two qubits when a SWAP gate is implemented. That is, the quantum gate consists of two qubits. As is clear from the figure, each input has one output. In the case of the first qubit, there are two inputs (00) and (01), corresponding to two outputs (10) and (11), and the same goes for the second qubit**.**

By comparing the results when using the two gates, the following becomes clear :

A SWAP gate is utilized to generate entangled states between two qubits, whereas a H gate is employed to generate entangled states on a single qubit. The SWAP gate changes the shape of the two qubits without changing their fundamental state. That is, the SWAP gate converts the state $|01\rangle$ to the state $|10\rangle$, and $|10\rangle$ to $|01\rangle$. While the H gate converts the state $|0\rangle$ to the state $(\alpha + \beta)/\sqrt{}2$, and the state of $|1\rangle$ to the state of $(\alpha - \beta)/\sqrt{2}$, where α and β are complex coefficients.

We may understand from programming quantum circuits consisting of 4-qubits that the simulation code explains the mechanism of interaction of the behavior of the qubits with the quantum gates in the quantum circuit. which gives the basic and necessary step to constructing or implementing any experimental work that requires the presence of four qubits with the outputs of 8 entangled states (maximally entangled) in the quantum system that the quantum computer will deal with.

5. Conclusion

The possibility of obtaining the same results for another type of input, such as $|1111\rangle$, but it is not specifically the same as the output state of the quantum circuit due to the difference in phase angle.

Other gates can be used in our quantum circuit, through which a superposition of the output state can be created, such as $R(\theta)$-gates instead of the H-gate. The Y-gate can be used instead of the X-gate, which is called qubit-flip and phase-flip.

Similar results can be given using gates that operate on three qubits, but the entangled state created by CNOT gates must be preserved. the Toffoli gate, which is a three-qubit gate with two controllers and a target, when placed in the quantum circuit randomly and not taking into account the $|q_0 q_1 q_2 q_3\rangle$ outputs may produce non-entangled states or Partially mixed or does not affect the circuit outputs. It is not possible to create 8 states for an output state resulting from a quantum circuit consisting of two qubits $|q_0 q_1\rangle$, but this can be created for a quantum circuit consisting of 3-qubits, and we also need three Hadamard gates, and this applies to $((2+n)\_qubits)$, where $n \geq 3$, represents the number of qubits in the circuit.

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