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RESEARCH ARTICLE - MATHEMATICS

On supra$^*\varrho_I$ – open sets , supra$^*\varrho_I$ – continuous functions

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Abstract

The concept of supra* ϱ_I-open sets (Sup* ϱ_I -o.Set) is presented and used to define the concepts of weakly supra*ϱ_I –closed (weakly Sup* ϱ_I -c.Set) , weakly supra*ϱ_I –continuous(W Sup* ϱ_I-Cont.), weakly supra*ϱ_I –neighborhood(W Sup* ϱ_I.neigh), and weakly supra*ϱ_I –irresolute(W Sup* ϱ_I.Irres). A few characterizations and features of these concepts are covered .

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1. Introduction

Kuratowski [1] and Vaidyanathaswamy [2] have both researched the topic of ideals in topological spaces. Additionally, Jankovic and Hamlett [3] looked into the characteristics of ideal topological spaces. A.S.Mashhour et al. [4] introduced supra topological spaces in 1983. El-Sheikh [5] investigated the characteristics of supra topological space and developed the idea of supra closure operator, which is produced by a supra topological spaces. In 2012, S. SEKAR et al. [6] introduced the idea of supra ID-open sets and supra ID-continuous functions and looked into a number of their properties. A new supra topology from an old via ideal was introduced via Ali Kandil et al. in 2015 [7]. We have also talked about the characteristics of this supra topology. In this paper, we introduced the idea of weakly supra* ϱ_I -closed sets, weakly supra* ϱ_I -continuous sets, weakly supra*ϱ_I-neighborhood sets, and weakly supra*ϱ_I - irresolute sets. A few characterizations and properties of these concepts are discussed.

Definition 2.1:[1]
A collection that isn't empty is ideal. If the ID of a subset of $\mathcal{H}$ meets both of the following two requirements, it is said to be an Ideal on $\mathcal{H}$.
(1) When $O \in ID$ and $P \subseteq O$, $P \in ID$ is implied.
(2) When $O \in ID$ and $P \in ID$, $O \cup P \in ID$ is implied.

Definition 2.2:[4]: A subset S for P(H) of non-empty set H be named a **supra topology on H** if S satisfy conditions :

1. S has $\mathcal{H}$ and $\emptyset$ in it.

2. S is closed under the arbitrary union

Supra topological space refers to the pair ($\mathcal{H}$,Sup). The member of Sup is known as the supra open set(Sup o.Set) in ($\mathcal{H}$,Sup) in this sense.

Definition 2.3:[4] Let Sup be the supra topology on $\mathcal{H}$ and let ($\mathcal{H}$,τ) be a topological space. If τ⊆Sup, we referred to Sup as a supra topology related to τ.

Definition 2.4:[8] Assume that Ʒ is a subset of a supra topological space ($\mathcal{H}$, Sup). The supra kernel of Ʒis the set {U ∈ Sup | Ʒ ⊂ U}, and it is represented by Sup-ker (Ʒ).

Defintion2.5: [8] Let (H,S) be supra topology spaces and K⊆H .hence

1.cl_{sup}(K)=∩{F⊆H: F is supra closed set and K⊆F} is named the **supra-closure** of K∈P(H).

2.int_{sup}=∪{C⊆X:C is supra open set and C⊆K} is named the **supra-interior** of K∈P(H)

Definition 2.6:[9] The ($\mathcal{H}$,Sup,ID) is called **ideal supra topological space(**denoted by**, ID SUP TS)** if (X,Sup)supra topological space and ID is ideal on X

Definition2.7: [9] Assume that($\mathcal{H}$,Sup,ID) be ideal supra topological space, Ʒ is subset of ($\mathcal{H}$,Sup,ID),then Ʒ is called to be **Supra Semi$_{ID}$*-open set(**denoted by **,Sup S$_{ID}$*o.Set)** if and only if Ʒ ⊂ $Cl_{sup}(Int^*_{sup}$(Ʒ)) and a subset Ʒ is called supra semi $_{ID}$* -close if it is complement is supra semi $_{ID}$* -opened.

Definition 2.8:[10] A function f:($\mathcal{H}_1$,Sup$_1$,ID)→($\mathcal{H}_2$,Sup$_2$) is called **supra Semi$_{ID}^*$-continuous(**denoted by, **Sup S$_{ID}$*-cont.)** if and only if the opposite image of any supra open set(Sup o.Set) in $\mathcal{H}_2$ is supra semi$_{ID}$-opened set(Sup S$_{ID}$-o.Set) in $\mathcal{H}_1$.

Definition 2.9:[11] Aset Ʒ is supra P$_I$- open (Sup P$_I$-o.Set) if Ʒ ⊂$Int^*_{sup}(Cl_{sup}$(Ʒ)). The complement of supra P_I^*-open is called supra P_I^*-close (Sup P$_I$*-c.Set).

Definition 2.10:[11] Let ($\mathcal{H}_1$, τ, ID) be ideal topological space(ID TS) and ($\mathcal{H}_2$, τ') be topological space(TS), Sup be an correlating supratopology with τ. **supra** P_I^***-continuous function(**denoted by **,Sup P$_I$-cont.)** is the function f: ($\mathcal{H}_1$, Sup, ID) → ($\mathcal{H}_2$, τ'). if the opposite image of any open set(o.Set) in $\mathcal{H}_2$ is a supra P$_I$- open set(Sup P$_I$-o.Set) in $\mathcal{H}_1$.

Definition 2.11: [12], [13] Let ($\mathcal{H}$, Sup, ID) be an ideal supra topological space(ID Sup TS) and Ʒ⊆ $\mathcal{H}$. if Ʒ ⊆ $Cl_{sup}(Int_{sup}(Cl^*_{sup}$(Ʒ)).then Ʒ is called is **supra β$_I$-open set(**denoted by **,Sup β$_I$-o.Set).**The complemented of supra β$_I$-open set is said supra β$_I$-close **(**denoted by **,Sup β$_I$-c.Set)**.

Definition2.12: [12], [14] Let ($\mathcal{H}_1$, τ, ID) be ideal topological space(ID Ts) and ($\mathcal{H}_2$, τ') be topological space, Sup be an correlating supra topology with τ. f: ($\mathcal{H}_1$, Sup, ID) → ($\mathcal{H}_2$, τ') is called **supra β $_I$ -continuous function(**denoted by**,Sup β$_I$-cont. F)** if and only if the opposite image of any open set(o.Set) in $\mathcal{H}_2$ is supraβ $_I$ -open set(Sup β$_I$-o.Set) in $\mathcal{H}_1$.

3. supra* ϱ_I-open set

Definition 3.1: A set Ʒ is supra*ϱ$_I$-open set(denoted by, **Sup*ϱ$_I$-o.Set**) if Ʒ ⊆ $Cl_{sup}(Int^*_{sup}$(Ʒ))∪ $Int_{sup}(Cl^*_{sup}$(Ʒ). The complement of supra*ϱ$_I$-open set is called supra*ϱ$_I$-close. The class of supra* ϱ$_I$-open set in X be indicated by Sup* ϱ$_I$O ($\mathcal{H}$,Sup,ID).

Example 3.2: Let $\mathcal{H}$={ c$_1$, c$_2$, c $_3$, c$_4$} with a supra Sup={ $\mathcal{H}$, ∅, { c $_2$}, {c $_1$, c $_3$}, {c $_1$,c $_2$, c $_3$}} , ID={∅, { c $_2$}} . Then the set Ʒ ={ c $_2$, c $_3$} is supra* ϱ$_I$-open.

Proposition 3.3: Let $\mathcal{Z}$ is a supra* ϱ_I-open such that $\mathrm{Int}^*_{\sup}(\mathcal{Z})=\emptyset$,then $\mathcal{Z}$ is supra P_I*-open. The following is true for a subset of Ideal supra topological space:

1.Any supra S_I* –open is also supra* ϱ_I- open.

2. Any supra P_I*-open is also supra* ϱ_I-open,

3. Any supra* ϱ_I-open is also supra β_I–open.

Proof:

(1)and(2)Obvious.

(3)Assume $\mathcal{Z}$ be supra* ϱ_I-opened. Next we have

$\mathcal{Z} \subseteq \mathrm{Cl}_{\sup}(\mathrm{Int}^*_{\sup}(\mathcal{Z}))\cup \mathrm{Int}_{\sup}(\mathrm{Cl}^*_{\sup}(\mathcal{Z}))$.

$\subseteq \mathrm{Cl}_{\sup}(\mathrm{Int}_{\sup}(\mathrm{Int}^*_{\sup}(\mathcal{Z}))\cup \mathrm{Int}_{\sup}(\mathrm{Int}_{\sup}\left(\mathrm{Cl}^*_{\sup}(\mathcal{Z})\right)$.

$\subseteq \mathrm{Cl}_{\sup}(\mathrm{Int}_{\sup}(\mathrm{Int}^*_{\sup}(\mathcal{Z}))\cup \mathrm{Int}_{\sup}\left(\mathrm{Cl}^*_{\sup}(\mathcal{Z})\right)$.

$\subseteq \mathrm{Cl}_{\sup}[\mathrm{Int}_{\sup}(\mathrm{Int}^*_{\sup}(\mathcal{Z}))\cup \mathrm{Cl}_{\sup}(\mathcal{Z})$.

$\subseteq \mathrm{Cl}_{\sup}[\mathrm{Int}_{\sup}[\mathrm{Cl}^*_{\sup}(\mathcal{Z}\cup \mathcal{Z}))]$

$\subseteq \mathrm{Cl}_{\sup}(\mathrm{Int}_{\sup}(\mathrm{Cl}^*_{\sup}(\mathcal{Z})))$.

This show that $\mathcal{Z}$ is supra β_I-open set .

Example 3.4: Let $\mathcal{H}$={ c_1, c_2, c_3, c_4} with a supra Sup={ $\mathcal{H}$, $\emptyset$, { c_2}, {c_1, c_3}, {c_1,c_2, c_3}} , ID={$\emptyset$, { c_2}} . Then the set $\mathcal{Z}$ ={ c_2, c_3} is supra* ϱ_I-open, but is not supra S_I^*-open. As $\mathrm{Cl}_{\sup}(\mathrm{Int}^*_{\sup}(\mathcal{Z})) \cup \mathrm{Int}_{\sup}(\mathrm{Cl}^*_{\sup}(\mathcal{Z}))=\mathrm{Cl}_{\sup}(\emptyset)\cup \mathrm{Int}_s(H) =\emptyset \cup \mathcal{H}= \mathcal{H} \supseteq \mathcal{Z}$ and hence $\mathcal{Z}$ is supra* ϱ_I-opened . Since $\mathrm{Cl}_{\sup}(\mathrm{Int}^*_{\sup}(\mathcal{Z})) = \mathrm{Cl}_{\sup}(\emptyset)=\emptyset \not\supseteq\mathcal{Z}$. Then $\mathcal{Z}$ is not supra S_I*-open.

Example 3.5: Let $\mathcal{H}$ ={ ι_1, ι_2, ι_3} with a supra Sup={ $\mathcal{H}$,$\emptyset$,{ ι_1},{ ι_3},{ ι_1, ι_3}} , ID={$\emptyset$,{ ι_2}} . Then the set $\mathcal{Z}$={ ι_1, ι_2} is supra* ϱ_I-opened , but is not supra P_I^*-opened Because $\mathrm{Cl}_{\sup}(\mathrm{Int}^*_{\sup}(\mathcal{Z})) \cup \mathrm{Int}_{\sup}(\mathrm{Cl}^*_{\sup}(\mathcal{Z}))= \mathrm{Cl}_{\sup}(\{\iota_3\}) \cup \mathrm{Int}_{\sup}(\{\iota_1, \iota_2\})=\{\iota_2,\iota_3\}\cup\{ \iota_1\}=\mathcal{H} \supseteq \mathcal{Z}$ and hence $\mathcal{Z}$ is supra* ϱ_I-opened . Since $\mathrm{Int}^*_{\sup}(\mathrm{Cl}_{\sup}(\mathcal{Z})) = \mathrm{Int}^*_{\sup}(\{\iota_1, \iota_2\})=\{ \iota_3 \} \not\supseteq \mathcal{Z}$ .Then $\mathcal{Z}$ is not supra P_I^{*}-opened.

Example 3.6: Let $\mathcal{H}$ ={ ι_1, ι_2, ι_3, ι_4} with a supra Sup={ $\mathcal{H}$, $\emptyset$, { ι_3},{ ι_1, ι_4},{ ι_1, ι_3, ι_4}}, ID= {$\emptyset$,{ ι_2}}. Then the set $\mathcal{Z}$= { ι_1, ι_2} is supra β_I-open, but is not supra* ϱ_I–open. Because $\mathrm{Cl}_{\sup}(\mathrm{Int}^*_{\sup}(\mathcal{Z})) \cup \mathrm{Int}_{\sup}(\mathrm{Cl}^*_{\sup}(\mathcal{Z}))= \mathrm{Cl}_{\sup}(\emptyset)\cup \mathrm{Int}_{\sup}(\{\iota_1 ,\iota_2, \iota_4\})= \emptyset \cup\{ \iota_1,\iota_4\}= \{ \iota_1, \iota_4\} \not\supseteq \mathcal{Z}$ and hence $\mathcal{Z}$ is not supra* ϱ_I-open . Since $\mathrm{Cl}_{\sup}(\mathrm{Int}_{\sup}(\mathrm{Cl}^*_{\sup}(\mathcal{Z})) = \mathrm{Cl}_{\sup}(\mathrm{Int}_{\sup} (\{ \iota_1, \iota_2, \iota_3\})= \mathrm{Cl}_{\sup}(\{ \iota_1,\iota_4\})=\{ \iota_1, \iota_2, \iota_3\}\supseteq \mathcal{Z}$. Hence $\mathcal{Z}$ is supra β_I –open.

Theorem3.7 : Asubset $\mathcal{Z}$ of an Ideal supra topological space ($\mathcal{H}$,Sup,ID) is supra* ϱ_I-close ,then $\mathrm{Cl}_{\sup}(\mathrm{Int}^*_{\sup}(\mathcal{Z}))\cap \mathrm{Int}_s(\mathrm{Cl}^*_{\sup}(\mathcal{Z}))\subseteq \mathcal{Z}$.

Proof: $\mathcal{H} - \mathcal{Z}$ is supra* ϱ_I-open ,because $\mathcal{Z}$ is supra* ϱ_I-closed, This can be shown from the fact that τ^* is finer than τ and the fact that we have

$\mathcal{H} - \mathcal{Z} \subseteq \mathrm{Cl}_{\sup}(\mathrm{Int}^*_{\sup}(\mathcal{H} - \mathcal{Z}))\cup \mathrm{Int}_{\sup}(\mathrm{Cl}^*_{\sup}(\mathcal{H} - \mathcal{Z}))$

$\subseteq \ Cl_{sup}(Int_{sup}(\mathcal{H}-Ȥ)) \cup Int_{sup}(Cl_{sup}\ (\mathcal{H}-Ȥ))$

$= [\mathcal{H}-[\ Cl_{sup}(Int_{sup}(\mathcal{H}-Ȥ))]] \cup [\mathcal{H}-[Int_{sup}(Cl_{sup}\ (\mathcal{H}-Ȥ))]]$

$\subseteq [\mathcal{H}-[\ Cl_{sup}(Int^{*}_{sup}(\mathcal{H}-Ȥ))]] \cup [\mathcal{H}-[Int_{sup}(Cl^{*}_{sup}\ (\mathcal{H}-Ȥ))]]$

$= \mathcal{H}-[\ Cl_{sup}(Int^{*}_{sup}(\mathcal{H}-Ȥ))]] \cup [\mathcal{H}-[Int_{sup}(Cl^{*}_{sup}\ (\mathcal{H}-Ȥ))]]$.

Therefore, we obtain $Cl_{sup}(Int^{*}_{sup}(Ȥ)) \cap Int_{sup}(Cl^{*}_{sup}(Ȥ)) \subseteq Ȥ$.

Corollary3.8: Let Ȥ represent a subset of the ideal supra topological space ($\mathcal{H}$,Sup,ID) suchthat$\mathcal{H}-[Cl_{sup}(Int^{*}_{sup}(Ȥ))] = Int_{sup}(Cl^{*}_{sup}(\mathcal{H}-Ȥ))$and $\mathcal{H}-[Int_{sup}(Cl^{*}_{sup}(Ȥ))] = Cl_{sup}(Int^{*}_{sup}(\mathcal{H}-Ȥ))$. Then Ȥ is supra* ϱ_{I} -closed if and only if $[Cl_{sup}(Int^{*}_{sup}(Ȥ)) \cap Int_{sup}(Cl^{*}_{sup}(Ȥ))] \subseteq Ȥ$.

Proof: This is the direct result of Theorem (3.6).

$[Cl_{sup}(Int^{*}_{sup}(Ȥ)) \cap Int_{sup}(Cl^{*}_{sup}(Ȥ))] \subseteq Ȥ$.

Then

$\mathcal{H}-Ȥ \subseteq \ \mathcal{H}-[Cl_{sup}(Int^{*}_{sup}(Ȥ)) \cap Int_{sup}(Cl^{*}_{sup}(Ȥ))]$.

$\subseteq [\mathcal{H}-[Cl_{sup}(Int^{*}_{sup}(Ȥ))]] \cup [\mathcal{H}-[Int_{sup}(Cl^{*}_{sup}(Ȥ))]]$

$= Cl_{sup}(Int^{*}_{sup}(\mathcal{H}-Ȥ)) \cup Int_{sup}(Cl^{*}_{sup}\mathcal{H}-Ȥ))$

Thus $\mathcal{H}-Ȥ$ is supra* ϱ_{I}-open and hence Ȥ is supra* ϱ_{I}-closed.

Proposition 3.9: The union of any family of supra* ϱ_{I}-open sets is a supra* ϱ_{I}-open set.

Pr00f: Let $\{Ȥ_{\alpha}\ /\ \alpha \in \Delta\}$ be a family of supra* ϱ_{I}-open set,

$Ȥ_{\alpha} \subseteq Cl_{sup}\left(Int^{*}_{s}(Ȥ_{\alpha})\right) \cup Int_{sup}\left(Cl^{*}_{sup}(Ȥ_{\alpha})\right)$

Hence

$\cup_{\alpha}Ȥ_{\alpha} \subseteq \cup_{\alpha}[Cl_{sup}(Int^{*}_{sup}(Ȥ_{\alpha})) \cup Int_{sup}(Cl^{*}_{sup}(Ȥ_{\alpha}))]$

$\subseteq \ \cup_{\alpha}[Cl_{sup}(Int^{*}_{sup}(Ȥ_{\alpha}))] \cup \cup_{\alpha}[Int_{sup}(Cl^{*}_{sup}(Ȥ_{\alpha}))]$

$\subseteq [Cl_{sup}(\cup_{\alpha}(Int^{*}_{sup}(Ȥ_{\alpha}))] \cup [Int_{sup}(\cup_{\alpha}(Cl^{*}_{sup}(Ȥ_{\alpha}))]$

$\subseteq [Cl_{sup}(\cup_{\alpha}(Int^{*}_{sup}(Ȥ_{\alpha}))] \cup [Int_{sup}(\cup_{\alpha}(Cl^{*}_{sup}(Ȥ_{\alpha}))]$

$\subseteq [Cl_{sup}(Int^{*}_{sup}(\cup_{\alpha}Ȥ_{\alpha}))] \cup [Int_{sup}(Cl^{*}_{sup}(\cup_{\alpha}Ȥ_{\alpha}))]$.

$\cup_{\alpha}Ȥ_{\alpha}$ is supra* ϱ_{I}-open .

Remark 3.10: As the following example demonstrates, the intersection of even two supra* ϱ_{I}-open sets need not be supra* ϱ_{I}-open set.

Example3.11: $\mathcal{H}$ ={ ɩ₁, ɩ₂, ɩ₃, ɩ₄ } with a supra Sup={ $\mathcal{H}$,∅,{ ɩ₃},{ ɩ₃, ɩ₂, ɩ₄}} and ID={∅, { ɩ₁ }, { ɩ₄ },{ ɩ₁, ɩ₄}}.Then the set Ȥ ={ ɩ₁, ɩ₃} and B={ ɩ₁, ɩ₂} are supra*b_{I} -open , but $Ȥ \cap B$ = { ɩ₁} is not supra*b_{I}-open

Definition 3.12: Let Ȥ be a subset of $\mathcal{H}$.

1-supra*ϱ_{I}-closure of Ȥ is defined as the intersection of all supra* ϱ_{I}-close containing Ȥ and is denoted via Clsup*ϱ_{I} (Ȥ).

2-The supra* ϱ_{I}-interior of Ȥ is defined by the union of all supra* ϱ_{I}-open sets contained in Ȥ and denoted via Intsup* ϱ_{I} (Ȥ)

Remark 3.13:

1. Let Ɀ represent a subset of the ideal supra topological space ($\mathcal{H}$,Sup,ID) .after that Ɀ is supra* ϱ_I-closed if and only if Clsup* ϱ_I (Ɀ)= Ɀ,

2. Let Q represent a subset of the ideal supra topological space ($\mathcal{H}$,Sup,ID) . Then Q is supra* ϱ_I-open if and only if Intsup*ϱ_I(Q)=Q.

4-supra$^*\varrho_I$-continuous function

Definition 4.1 :A function f:($\mathcal{H}_1$,Sup_1,ID)⟶($\mathcal{H}_2$,Sup_2) is named supra*ϱ_I-continuous if f^{-1}(Ɀ) is supra*ϱ_I-open in $\mathcal{H}_1$. for every supra open set Ɀ of $\mathcal{H}_2$.

Example 4.2: Let $H_1 = H_2 = \{ ι_1, ι_2, ι_3 \}$, with two supra $S_2 = \{ H_1, φ, \{ ι_2\}\}$and

$S_2 = \{H_2, φ, \{ ι_1, ι_2\}\}$, and ID=$\{\emptyset, \{ ι_3\}\}$ be an ideal on X. Define a function f:(X,S_1,ID)⟶(X, S_2) . $f(ι_1) = ι_2$, $f(ι_2) = ι_1$ and $f(ι_3) = ι_3$. It is clear that f is supra*ϱ_I-continuous

Definition 4.3: Let Ɀ be a subset of a space ($\mathcal{H}$,Sup,ID) and let h ∈ $\mathcal{H}$. If there exist supra*ϱ_I-open set Q containing h such that Q⊆ Ɀ. After that Ɀ is called supra*ϱ_I-neighb0rhood of h.

Theorem4.4: for a function Ł:($\mathcal{H}_1$,Sup_1,ID) ⟶($\mathcal{H}_2$,Sup_2),the statements that follow are equivalent

1. Ł is supra*ϱ_I-continuous,

2.,there exist supra*ϱ_I-open set Q containing h such that Ł (Q)⊆ Ɀ, For every h∈ $\mathcal{H}_1$ and every supra open set Ɀ in $\mathcal{H}_2$ with Ł (h)∈ Ɀ

3. For each h∈ $\mathcal{H}_1$and each supra open set Ɀ in $\mathcal{H}_2$ with Ł (h)∈ Ɀ, $Ł^{-1}$(Ɀ) is supra*ϱ_I- neighb0rhood of h,

4. (Int^*_{sup}(*ϱ_I)(Ɀ))⊆ Sup-Ker(Ł (Ɀ)), For every subset Ɀ of $\mathcal{H}_1$,

5. For every subset E of $\mathcal{H}_2$, Int^*_{sup}(*ϱ_I)($Ł^{-1}$(E))⊆(Sup-Ker(E)).

Proof:

(1)⟹(2): Assume h∈ $\mathcal{H}_1$ and Lets Ɀ be a supra open set in $\mathcal{H}_2$ s.t Ł (h) ∈ Ɀ. Because Ł is supra* ϱ_I-continuous,$Ł^{-1}$(Ɀ) is supra*ϱ_I-open. By butting Q= $Ł^{-1}$(Ɀ) which is containing h , we have Ł (Q)⊆ Ɀ.

(2)⟹(3): Let Ɀ be a supra open set in $\mathcal{H}_2$ such that Ł (h) ∈ Ɀ.Then by (2) there exists supra*ϱ_I-open set Q containing h such that Ł (Q)⊆ Ɀ. So h ∈ Q⊆ $Ł^{-1}$(Ɀ). Hence $Ł^{-1}$(Ɀ) is supra*ϱ_I-neighborhood of h.

(3) ⟹(1) : Let Ɀ be a supra open set in $\mathcal{H}_2$ such that Ł (h)∈ Ɀ. Then by (3), $Ł^{-1}$(Ɀ) is supra*ϱ_I-neighborhood of h . Thus for each h∈ $Ł^{-1}$(Ɀ). There exists a supra*ϱ_I-open set Qh containing h such that h ∈Qh ⊆ $Ł^{-1}$(Ɀ).Hence$Ł^{-1}$(Ɀ) ⊆ $Q_{h\in Ł^{-1}(Ɀ)}$ and so $Ł^{-1}$(Ɀ) ∈Sup* ϱ_IO(X).

(1) ⟹(4): Let Ɀ be any subset of $\mathcal{H}_1$. Suppose that k ∉ Sup-Ker(Ɀ) . Then by lemma 2.5 , there exists a closed subset N of $\mathcal{H}_2$ such that k∈N and Ł(Ɀ)∩N =∅.Thus we have

Ɀ∩ $Ł^{-1}$(N)= ∅ and(Int^*_{sup}(*ϱ_I)(Ɀ)) ∩ $Ł^{-1}$(N)= ∅. Therefore, we obtain Ł(Int^*_{sup}(*ϱ_I)(Ɀ)) ∩(N) = ∅ and k ∉ N(Int^*_{sup}(*ϱ_I) (Ɀ)).This implies that Ł (Int^*_{sup}(*ϱ_I)(Ɀ))⊆Sup-Ker(Ł (Ɀ)).

(4) ⟹(5): Let E be any subset of $\mathcal{H}_2$. By (4) and lemma 2.5,

we have (Int^*_{sup}(*ϱ_I) ($Ł^{-1}$(E)))⊆ Sup-Ker(Ł($Ł^{-1}$(E)))⊆Sup-Ker(E)

and Int^*_{sup}(*ϱ_I)($Ł^{-1}$(E))⊆ $Ł^{-1}$(Sup-ker(E)).

(5) ⟹(1) : Let Ꝁ be any supra subset of $\mathcal{H}_2$. By (5) and lemma 2.5, we have $\mathrm{Int}^*_{\mathrm{sup}}(*\varrho_{\mathrm{I}})$ $(Ł^{-1}(Ꝁ))\subseteq Ł^{-1}$(Sup-Ker(Ꝁ))= $Ł^{-1}$(Ꝁ), $\mathrm{Int}^*_{\mathrm{sup}}(*\varrho_{\mathrm{I}})(Ł^{-1}(Ꝁ))= Ł^{-1}(Ꝁ)$. This shows that $Ł^{-1}$(Ꝁ) is supra*ϱ_{I}-open.

Definition 4.5: Let Đ is a subset of the ideal supra topological space ($\mathcal{H}$,Sup,ID) is said to be weakly supra* ϱ_{I}-open(denoted by **,** if $Đ \subseteq \mathrm{Cl}_{\mathrm{sup}}(\mathrm{Int}^*_{\mathrm{sup}}(\mathrm{Cl}_{\mathrm{sup}}(Đ)))\cup\mathrm{Cl}_{\mathrm{sup}}(\mathrm{Int}_{\mathrm{sup}}(\mathrm{Cl}^*_{\mathrm{sup}}(Đ)))$.

The class of weakly supra*ϱ_{I}-open set in $\mathcal{H}$ be indicated by WSup*ϱ_{I}O ($\mathcal{H}$,Sup,ID).

Proposition 4.6: For a subset of an ideal supra topological space, every supra*ϱ_{I}-open set is weakly supra*ϱ_{I}-open.

Proof: Let Ꝁ be a supra*ϱ_{I}-open set. Then $Ꝁ\subseteq \mathrm{Cl}_{\mathrm{sup}}(\mathrm{Int}^*_{\mathrm{sup}}(Ꝁ))\cup \mathrm{Int}_{\mathrm{sup}}(\mathrm{Cl}^*_{\mathrm{sup}}(Ꝁ) \subseteq \mathrm{Cl}_{\mathrm{sup}}(\mathrm{Int}^*_{\mathrm{sup}}(\mathrm{Cl}_{\mathrm{sup}}(Ꝁ)))\cup\mathrm{Cl}_{\mathrm{sup}}(\mathrm{Int}_{\mathrm{sup}}(\mathrm{Cl}^*_{\mathrm{sup}}(Ꝁ)))$.This shows that Ꝁ is a weakly supra*ϱ_{I}-open set .

As the following example demonstrates, the inverse of the aforementioned theorem need not be true.

Example 4.7 : $\mathcal{H}$ ={ ι_1, ι_2, ι_3, ι_4 } with supra Sup={ $\mathcal{H}$,∅,{ ι_3},{ ι_1, ι_4},{ ι_1, ι_2, ι_3}} and ID={∅, { ι_3}}. Then the set Ꝁ ={ ι_1, ι_2} is weakly supra*ϱ_{I}-open ,but it is not supra*ϱ_{I}-open.

Theorem 4.8 : Let ($\mathcal{H}$,Sup,ID) be an ideal supra topological space .If $\mathrm{U}_\alpha \in$ WSup*ϱ_{I}O($\mathcal{H}$) for eachα ∈ Δ,then $\cup\{\mathrm{U}_\alpha : \alpha \in \Delta\} \in$WSup*$\varrho_{\mathrm{I}}$O($\mathcal{H}$,Sup,ID).

Proof: Since U_α ∈WSup*ϱ_{I}O($\mathcal{H}$,Sup,ID) ,we have

$$\cup_{\alpha\in\Delta} \mathrm{U}_\alpha \subseteq \cup_{\alpha\in\Delta}\mathrm{Cl}_{\mathrm{sup}}(\mathrm{Int}^*_{\mathrm{sup}}(\mathrm{Cl}_{\mathrm{sup}}(\mathrm{U}_\alpha)))\cup\mathrm{Cl}_{\mathrm{sup}}(\mathrm{Int}_{\mathrm{sup}}(\mathrm{Cl}^*_{\mathrm{sup}}(\mathrm{U}_\alpha))).$$

$$\subseteq \cup_{\alpha\in\Delta}\mathrm{Cl}_{\mathrm{sup}}(\mathrm{Int}^*_{\mathrm{sup}}(\mathrm{Cl}_{\mathrm{sup}}(\cup_{\alpha\in\Delta} \mathrm{U}_\alpha)))\cup\mathrm{Cl}_{\mathrm{sup}}(\mathrm{Int}_{\mathrm{sup}}(\mathrm{Cl}^*_{\mathrm{sup}}(\cup_{\alpha\in\Delta} \mathrm{U}_\alpha))).$$

Hence $\cup_{\alpha\in\Delta}\mathrm{U}_\alpha$ ∈WSup*ϱ_{I}O($\mathcal{H}$,Sup,ID).

The finite intersection of weakly supra*ϱ_{I}-open sets does not necessarily have to be weakly supra*ϱ_{I}-open sets, as demonstrated by the example that follows.

Example 4.9 : $\mathcal{H}$={ ι_1, ι_2,ι_3, ι_4} with supra Sup={ $\mathcal{H}$,∅,{ι_3},{ι_3,ι_2,ι_4}} and ID={∅,{ ι_1},{ ι_4},{ ι_1, ι_4}} . Then the set Ꝁ ={ ι_1, ι_3} and B={ ι_1, ι_2} are weakly supra*ϱ_{I}-open, but Ꝁ∩B ={ ι_1} is not weakly supra*ϱ_{I}-open

Definition 4.10: A subset Ꝁ

of an ideal supra topological space ($\mathcal{H}$,Sup,ID) is said to be weakly supra*ϱ_{I}-closed if its complement is weakly supra*ϱ_{I}–open.

Theorem 4.11: If a subset Ꝁ of an ideal topological space ($\mathcal{H}$,Sup,ID) is said to be weakly supra*ϱ_{I}-closed , then $\mathrm{Int}_{\mathrm{sup}}(\mathrm{Cl}_{\mathrm{sup}}(\mathrm{Int}^*_{\mathrm{sup}}(Ꝁ)))\subseteq Ꝁ$.

Proof: Since Ꝁ is weakly supra*ϱ_{I}-closed, $\mathcal{H} - Ꝁ$ is weakly supra*ϱ_{I}–open. This mean, $\mathcal{H} - Ꝁ$ ∈WSup*ϱ_{I}O($\mathcal{H}$) by the fact $\mathrm{Sup}^* \subseteq \mathrm{Sup}^*_{\mathrm{I}}$, and the fact $\mathrm{Sup}^* \subseteq \mathrm{Sup}$ then,

$$\mathcal{H} - Ꝁ \subseteq \mathrm{Cl}_{\mathrm{sup}}(\mathrm{Int}^*_{\mathrm{sup}}(\mathrm{Cl}_{\mathrm{sup}}(\mathcal{H} - Ꝁ)))\cup \mathrm{Cl}_{\mathrm{sup}}(\mathrm{Int}_{\mathrm{sup}}(\mathrm{Cl}^*_{\mathrm{sup}}(\mathcal{H} - Ꝁ))))$$

$$\subseteq \mathrm{Cl}_{\mathrm{sup}}(\mathrm{Int}_{\mathrm{sup}}(\mathrm{Cl}_{\mathrm{sup}}(\mathcal{H} - Ꝁ))) \cup\mathrm{Cl}_{\mathrm{sup}}(\mathrm{Int}_{\mathrm{sup}}(\mathrm{Cl}^*_{\mathrm{sup}}(\mathcal{H} - Ꝁ))))$$

$$\subseteq \mathrm{Cl}_{\mathrm{sup}}\left[\mathrm{Int}_{\mathrm{sup}}\left(\mathrm{Cl}_{\mathrm{sup}}(\mathcal{H} - Z)\right)\cup\mathrm{Int}_{\mathrm{sup}}\left(\mathrm{Cl}^*_{\mathrm{sup}}(\mathcal{H} - Z)\right)\right]$$

$$\subseteq \mathrm{Cl}_{\mathrm{sup}}[\mathrm{Int}_{\mathrm{sup}}[\mathrm{Cl}_{\mathrm{sup}}(\mathcal{H} - Ꝁ) \cup\mathrm{Cl}^*_{\mathrm{sup}}(\mathcal{H} - Ꝁ)]]$$

$$\subseteq \mathrm{Cl}_{\mathrm{sup}}[\mathrm{Int}_{\mathrm{sup}}[(\mathrm{Cl}_{\mathrm{sup}}(\mathcal{H} - Ꝁ)) \cup\mathrm{Cl}^*_{\mathrm{sup}}(\mathcal{H} - Ꝁ)]]$$

$$\subseteq \mathrm{Cl}_{\mathrm{sup}}[\mathrm{Int}_{\mathrm{sup}}(\mathrm{Cl}^*_{\mathrm{sup}}(\mathcal{H} - Ꝁ))]$$

$$=\mathcal{H} - \mathrm{Int}_{\mathrm{sup}}(\mathrm{Cl}_{\mathrm{sup}}(\mathrm{Int}^*_{\mathrm{sup}}(Ꝁ)))$$

Therefore we get the result $\mathrm{Int}_{\mathrm{sup}}(\mathrm{Cl}_{\mathrm{sup}}(\mathrm{Int}^{*}_{\mathrm{sup}}(Z))) \subseteq Z$.

Definition 4.12: A function $\mathcal{F}:(\mathcal{H}_1,Sup_1,ID)\longrightarrow(\mathcal{H}_2,Sup_2)$ is said to be weakly supra*ϱ_I–continuous(denoted by **, W Sup*ϱ_I-Cont**) if for each supra open set Z of $(\mathcal{H}_2,Sup_2)$,$\mathcal{F}^{-1}(Z)$ is weakly supra*ϱ_I-open in $(\mathcal{H}_1,Sup_1,ID)$.

Remark 4.13: Every supra*ϱ_I-continuous is weakly supra*ϱ_I-continuous. The following example show that weakly supra*ϱ_I-continuous function do not need to be supra*ϱ_I-continuous.

Example 4.14: Let $\mathcal{H}=\{\iota_1,\iota_2,\iota_3,\iota_4\}$ with a supra $Sup_1=\{\mathcal{H},\emptyset,\{\iota_3\},\{\iota_3,\iota_4\},\{\iota_1,\iota_3,\iota_4\}\}$, $Sup_2=\{\mathcal{H},\emptyset,\{\iota_3,\iota_2\}\}$ and $ID=\{\emptyset,\{\iota_3\}\}$ .Then the identity function $f:(\mathcal{H},Sup_1)\longrightarrow(\mathcal{H},Sup_2)$ is weakly supra*ϱ_I-continuous but it is not supra*ϱ_I-continuous .

Proposition 4.15: For a function $f:(\mathcal{H}_1,Sup_1,ID)\longrightarrow(\mathcal{H}_2,Sup_2)$,the following statements are equivalent :

(1)f is weakly supra*ϱ_I-continuous,

(2)For any $h\in\mathcal{H}_1$ and $V\in Sup_2$ with $f(h)\in V$, there exist $Q\in WSup^*\varrho_I O(\mathcal{H},Sup,ID)$ with $h\in Q$ such that $f(Q)\subseteq V$,

(3) The inverse image of every supra closed set in $\mathcal{H}_2$ is weakly supra*ϱ_I-close in $\mathcal{H}_1$.

Proof: Straightforward

Definition 4.16: Let E is a subset of a space $(\mathcal{H},Sup,ID)$ and let $h\in\mathcal{H}$. Then E is called a weakly supra*ϱ_I-neighborhood (denoted by, **W Sup* ϱ_I.neigh**) of h if there exists a weakly supra*ϱ_I-open set Q containing h such that $Q\subseteq E$.

Example 4.17 : $\mathcal{H}=\{\iota_1,\iota_2,\iota_3,\iota_4\}$ with supra $Sup=\{\mathcal{H},\emptyset,\{\iota_3\},\{\iota_3,\iota_2,\iota_4\}\}$ and $ID=\{\emptyset,\{\iota_1\},\{\iota_4\},\{\iota_1,\iota_4\}\}$. Then the set $E=\{\iota_1,\iota_3,\iota_4\}$ is weakly supra*ϱ_I-neighborhood ,Since $Q=\{\iota_1,\iota_3\}$ is supra*-open set s.t $Q\subseteq E$.

Example 4.18 : $\mathcal{H}=\{\iota_1,\iota_2,\iota_3,\iota_4\}$ with supra $Sup=\{\mathcal{H},\emptyset,\{\iota_3\},\{\iota_3,\iota_2,\iota_4\}\}$ and $ID=\{\emptyset,\{\iota_1\},\{\iota_4\},\{\iota_1,\iota_4\}\}$.

$E=\{\iota_1\}$ is not weakly supra*ϱ_I-neighborhood Since there is not exist supra*-open set subset of E .

Theorem 4.19: For a function $f:(\mathcal{H}_1,Sup_1,ID)\longrightarrow(\mathcal{H}2,Sup_2)$,the following statements are equivalent :

(1)f is weakly supra* ϱ_I-continuous.

(2)For each $h\in\mathcal{H}_1$ and each supra open set V in $\mathcal{H}_2$ with $f(h)\in V$,$f^{-1}(V)$ is weakly supra* ϱ_I -neighborhood of h .

Proof:

(1)⟹(2) : Let $h\in\mathcal{H}_1$ and let V be a supra open set in $\mathcal{H}_2$ such that $f(h)\in V$. By Propostion 4.14, there exists a weakly supra*ϱ_I-open Q in $\mathcal{H}_1$ with $h\in Q$ such that $f(Q)\subseteq V$ .So $h\in Q\subseteq f^{-1}(V)$. Hence $f^{-1}(V)$ is a weakly supra*ϱ_I-neighborhood of h.

(2) ⟹(1) : Let V be a supra open in $\mathcal{H}_2$ and let $f(h)\in V$. Then by assumption $f^{-1}(V)$ is a weakly supra*ϱ_I-neighborhood of h. Thus for each $h\in f^{-1}(V)$

there exists a weakly supra*ϱ_I-open set Qh containing h such that $h\in Q_h\subseteq f^{-1}(V)$. Hence $f^{-1}(V)=\cup\{Qh : h\in f^{-1}(V)\}$ and so $f^{-1}(V)\in WSup^*\varrho_I O(\mathcal{H}_1,Sup_1,ID)$.

Definition 4.20: A function f:($\mathcal{H}_1$,Sup_1,ID) ⟶($\mathcal{H}_2$,Sup_2) is said to be weakly supra*ϱ_I-irresolute(denoted by ,**W Sup* ϱ_I.Irres**) if $f^{-1}(V) \in WSup^*\varrho_I O(\mathcal{H}_1)$ for every $V \in WSup^*\varrho_I O(\mathcal{H}_2)$.

Example 4.2: Let $H_1 = H_2 = \{ \iota_1, \iota_2, \iota_3 \}$, with two supra $S_2 = \{ H_1, \varphi \}$and

$S_2 = \{H_2, \varphi, \{ \iota_1, \iota_2\}\}$, and ID=$\{\emptyset, \{ \iota_1\},\{ \iota_4\},\{ \iota_1, \iota_4\}\}$ be an ideal on H_1 and H_2. Define a function f:(X,S_1,ID)⟶(X, S_2 ,ID) . $f(\iota_1) = \iota_2$, $f(\iota_2) = \iota_1$ and $f(\iota_3) = \iota_3$. It is clear that f is weakly supra*ϱ_I- irresolute.

Example 4.21: Let $H_1 = H_2 = \{ \iota_1, \iota_2, \iota_3 \}$, with two supra $S_1 = \{ \mathcal{H}, \emptyset, \{\iota_3\}, \{\iota_3, \iota_2, \iota_4\}\}$ and

$S_2 = \{ \mathcal{H}, \emptyset, \{ t_1, t_2\}\}$, and ID=$\{\emptyset, \{ \iota_3\}\}$ be an ideal on on $H_1$and H_2 . Define a function f:(X,S_1,ID)⟶(X, S_2,ID) . $f(\iota_1) = \iota_1$, $f(\iota_2) = \iota_1$ and $f(\iota_3) = \iota_3$. It is clear that f is not

weakly supra*ϱ_I- irresolute.

Theorem 4.22: Let f:($\mathcal{H}_1$,Sup_1,ID)⟶(Y,Sup_2,ID) and g:($\mathcal{H}_2$,Sup_2,ID') ⟶($\mathcal{H}_3$, ID")be two function Then:

(1) g ∘ f is weakly supra*ϱ_I-continuous if f is weakly supra*ϱ_I-irresolute and g is weakly supra*ϱ_I-continuous.

(2) g ∘ f is weakly supra*ϱ_I-continuous if f is weakly supra*ϱ_I-continuous and g is continuous.

Proof:

(1) Let h∈ $\mathcal{H}_1$ and W be any supra open set of Z containing (g ∘ f)(h). Since g is weakly supra*ϱ_I-continuous, there exists $V \in WSup^*\varrho_I O(\mathcal{H}_2)$ such that f(h) ∈V and g(V)⊆W. Again, since f is weakly supra*ϱ_I-irresolute, there exists $Q \in WSup^*\varrho_I O(\mathcal{H}_1,h)$ such that f(Q)⊆V.This shows that (g ∘ f)(Q) ⊆W. Hence g ∘ f is weakly supra*ϱ_I-continuous.

(2)Let h∈ $\mathcal{H}_1$ and W be any supra open set of Z containing (g ∘ f)(h). Since g is continuous, V $=g^{-1}(W)$ is open in $\mathcal{H}_2$, Also, since f is weakly supra*ϱ_I-continuous , there exists $Q \in WSup^*\varrho_I O(\mathcal{H}_1,\tau)$ such that h ∈Q and f(Q) ⊆ V. Therefore (g ∘ f)(Q) ⊆W.Hence g ∘ f is weakly supra*ϱ_I-continuous.

CONCLUSION

In this paper, we have presented supraϱ_I--open set with respect to an ideal (briefly supraϱ_I-closed set) in supratopological spaces. We characterized variants of continuity namely supraϱ_I-continuous, weakly supra ϱ_I—continuous, weakly supra ϱ_I—irresolute.

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