Mustansiriyah Journal of Pure and Applied Sciences, Print ISSN: 2957-9910. Online ISSN: 2957-9929, Vol. 3, No. 2, 2025, Pages 32-40
DOI: 10.47831/mjpas.v3i2.186

MJPAS

MUSTANSIRIYAH JOURNAL OF PURE AND APPLIED SCIENCES

Journal homepage:
https://mjpas.uomustansiriyah.edu.iq/index.php/mjpas

RESEARCH ARTICLE - PHYSICS

Initial Spot Size Influence of Hollow Gaussian Laser Beam on the Self-Focusing Phenomenon inside Magnetized Plasma

Ameer Ali Hassan [1, 2 *], Abdul Kareem H. Dagher [1], Munther B. Hassan [3]

[1]Department of Physics, Faculty of Education, Al-Mustansiriyah University, Baghdad, Iraq

[2]Department of Medical Physics, Faculty of Medical Sciences, Jabir Ibn Hayyan Medical University, Najaf, Iraq

[3]Department of Physics, Faculty of Science, University of Kufa, Najaf, Iraq

[*] Corresponding author E-mail: ameer.a.hassan@jmu.edu.iq

Article Info.

Article history:

Received
18 April 2024

Accepted
26 May 2024

Publishing
30 March 2025

Abstract

This article presents a theoretical study of the self-focusing process of a hollow Gaussian laser beam (HGLB) inside a plasma medium placed in a magnetic field perpendicular to the direction of the hollow Gaussian laser beam propagation. The self-focusing was studied using relativistic nonlinearity. Particular mathematical equations were derived that describe the nonlinear behavior of hollow Gaussian laser beam when they propagate inside the plasma, and they were solved. This behavior was simulated using the MATLAB program. This study dealt with the effect of Initial spot size of hollow Gaussian laser beam on the self-focusing process.

The official journal published by the College of Education at Mustansiriyah University

Keywords: Self-focusing Phenomenon, Spot size, Hollow Gaussian laser beam, Relativistic Nonlinearity.

1. Introduction

Interest in nonlinear phenomena has recently increased, whether experimentally or theoretically simulated, because of its broad results and applications [1-6]. One of the nonlinear phenomena researchers are interested in is the production of stimulated positive lenses produced by self-focusing of the laser beam inside the plasma [7, 8]. As a result of the high intensity of the laser beam provided by the self-focusing phenomenon, it is used in critical applications such as generating terahertz radiation, separating uranium isotopes, ion accelerators, and nuclear fusion reactors [9-11]. Laser beams have many modes, the most commonly used form of which is the central mode (Gaussian mode), and there is limited use for the rest of the modes, such as hollow Gaussian laser beam, Hermite- Gaussian laser beam (H-GLB), and cosh-Gaussian laser beam (C-GLB) [12-14]. The self-focusing of the laser beam occurs through three types of nonlinearities depending on the laser pulse length.

These types are Ohmic nonlinearity, ponderomotive nonlinearity, and relativistic nonlinearity [15-17]. In the case of an applied Magnetic field parallel to the direction of wave propagation, the plasma medium will act as a circularly polarized plate. The dielectric constant will change depending on the applied field. When the order in which wave propagation is perpendicular to the magnetic field. the situation will differ from the first case. The dielectric constant will change differently depending on the vertical field [18,19]. In this paper, we study the effect of Initial Spot Size on the relativistic self-focusing of Hollow Gaussian Laser Beam (HGLB) propagating in magnetoplasma under paraxial approximation. Hollow Gaussian Laser Beam propagation is perpendicular to the direction of an external transverse magnetic field. The article's structure will be as follows: In section (2), the relativistic nonlinear dielectric tensors are derived. Section (3) contains the equations for the self-focusing of the hollow Gaussian laser beam. The numerical results are introduced along with a discussion of the overall findings and conclusions in Sections (4) and (5), respectively.

2. Plasma dielectric tensor in relativistic nonlinearity

Consider a HGLB inside a uniform magnetized plasma with a Transverse Magnetic Field aligned along $\hat{z}$– direction$\vec{B}_0||\hat{z}$. AHollow Gaussian Laser Beam propagates through it along the $\hat{x}$- path. The Electric field vector $\vec{E}_0$ of hollow Gaussian Laser Beam wave propagating in $\hat{y}$ – direction through the magnetoplasma can be written as follows [20]:

$$\vec{E}_0 = \vec{A}_0 e^{i(\omega_0 t - kx)} \qquad (1)$$

Where: $\vec{A}_0 = \hat{x}A_x + \hat{y}A_y$ is the amplitude of the electric field, k, and $\omega_0$are wave vector and the angular frequency respectively. The response of plasma electrons for Hollow Gaussian Laser Beamis controlled by the equation of motion for relativity. [21]:

$$m_0\gamma \frac{\partial}{\partial t}\vec{\upsilon}_0 = -e\vec{E}_0 - \frac{e}{c}\left(\vec{\upsilon}_0 \times \vec{B}_0\right) \qquad (2)$$

Where: $\vec{\upsilon}_0$ represents the velocity of oscillation of plasma electrons gained by laser beam, **c** is the velocity of light and γ is the relativistic factor (the Lorentz factor), also m_0 , $-e$ represents rest mass of electron and electron charge, respectively. By replacing $\frac{\partial}{\partial t}$ with $-i\,\omega_0$ and, $\omega_{ce0} = \frac{eB_0}{m_0 c}$,we can rewrite Eq.(2) as follows [22]:

$$-i\,\omega_0\gamma\vec{\upsilon}_0 = \frac{-e}{m_0}\vec{E}_0 - \left(\vec{\upsilon}_0 \times \omega_{ce0}\right) \qquad (3)$$

Where: ω_{ce0} , ω_0 are the angular frequencies of electron cyclotron and Hollow Gaussian Laser Beam, respectively ; from Eq. (3), One could obtain the velocity components υ_{0x} and υ_{0y} of the electron as [22]:

$$\upsilon_{0x} = \frac{e\,\omega_{ce0}}{\gamma^2 m_0\omega_0^2\left(-1+\frac{\omega_{ce0}^2}{\gamma^2\omega_0^2}\right)}E_y + \frac{ie}{\gamma m_0\omega_0\left(-1+\frac{\omega_{ce0}^2}{\gamma^2\omega_0^2}\right)}E_x \qquad (4a)$$

$$\upsilon_{0y} = \frac{ie}{\gamma m_0\omega_0\left(-1+\frac{\omega_{ce0}^2}{\gamma^2\omega_0^2}\right)}E_y - \frac{e\,\omega_{ce0}}{\gamma^2 m_0\omega_0^2\left(-1+\frac{\omega_{ce0}^2}{\gamma^2\omega_0^2}\right)}E_x \qquad (4b)$$

Relativistic factor γ can be written as:

$$\gamma = \left(1+\frac{\upsilon_0^2}{2c^2}\right) \qquad (5)$$

Using Eq.(4), one may obtain the current density $\vec{J} = \sigma\vec{E} = -n_0 e\vec{\upsilon}_0$, where σ representing the conductivity tensor. The dielectric constants $\varepsilon = I + \frac{4\pi\sigma}{\omega_0}$. The dielectric constant tensor's components in the relativistic regime will be as follows:

$$\varepsilon = \begin{vmatrix} \varepsilon_{xx} = 1-\frac{\omega_{pe0}^2}{\omega_0^2\gamma\left(1-\frac{\omega_{ce0}^2}{\omega_0^2\gamma^2}\right)} & \varepsilon_{xy} = \frac{i\left(\frac{\omega^2_{pe0}}{\omega_0^2\gamma}\right)\left(\frac{\omega_{ce0}}{\omega_0\gamma}\right)}{\left(1-\frac{\omega_{ce0}^2}{\omega_0^2\gamma^2}\right)} & \varepsilon_{xz} = 0 \\ \varepsilon_{yx} = \frac{-i\left(\frac{\omega^2_{pe0}}{\omega_0^2\gamma}\right)\left(\frac{\omega_{ce0}}{\omega_0\gamma}\right)}{\left(1-\frac{\omega_{ce0}^2}{\omega_0^2\gamma^2}\right)} & \varepsilon_{yy} = 1-\frac{\omega_{pe0}^2}{\omega_0^2\gamma\left(1-\frac{\omega_{ce0}^2}{\omega_0^2\gamma^2}\right)} & \varepsilon_{yz} = 0 \\ \varepsilon_{zx} = 0 & \varepsilon_{zy} = 0 & \varepsilon_{zz} = 1-\frac{\omega_{pe0}^2}{\omega_0^2\gamma} \end{vmatrix} \qquad (6)$$

Where $\omega_{pe0} = \left(\frac{4\pi n_0 e^2}{m_0}\right)^{\frac{1}{2}}$ is the frequency of electron plasma, and n_0 is the background plasma density. From Maxwell's equation $\vec{\nabla}\cdot D = \vec{\nabla}\cdot(\varepsilon_{eff}\vec{E}) = 0$ [21], one obtains

$$E_y \approx \frac{-\varepsilon_{yx}}{\varepsilon_{yy}} E_x \quad (7) \qquad \text{and} \qquad \varepsilon_{eff} = \frac{\varepsilon_{xx}^2 + \varepsilon_{xy}^2}{\varepsilon_{xx}} \quad (8)$$

Where ε_{eff} is the effective dielectric constant for the relativistic case. By putting Eq.(6) in Eq.(8) can write the effective dielectric constant for relativistic case as follows:

$$\varepsilon_{eff} = 1 - \frac{\frac{\omega_{pe0}^2}{\gamma}\left(\omega_0^2 - \frac{\omega_{pe0}^2}{\gamma}\right)}{\omega_0^2\left(\omega_0^2 - \frac{\omega_{ce0}^2}{\gamma^2} - \frac{\omega_{pe0}^2}{\gamma}\right)} \quad (9)$$

To the zeroth order, one may take γ = 1. If a magnetic field is supplied in an orientation that is perpendicular to the direction of wave propagation. the electron oscillation velocity will change as follows:

$$\upsilon_0 = \frac{1}{\sqrt{2}}\left(\upsilon_{0x}\cdot\upsilon_{0x}^* + \upsilon_{0y}\cdot\upsilon_{0y}^*\right)^{\frac{1}{2}} \quad (10)$$

By putting Eq.(4) in Eq.(10) and using Eq.(7), we can rewrite Eq.(5) as follows:

$$\gamma = 1 + \frac{1}{4}\left(\frac{e}{m_0\omega_0 c}\right)^2\left[1 + 3\left(\frac{\omega_{ce0}}{\omega_0}\right)^2 + 4\left(\frac{\omega_{ce0}}{\omega_0}\right)^2\left(\frac{\omega_{pe0}}{\omega_0}\right)^2 + 2\left(\frac{\omega_{ce}0}{\omega_0}\right)^4 + 5\left(\frac{\omega_{ce0}}{\omega_0}\right)^2\left(\frac{\omega_{pe0}}{\omega_0}\right)^4 + 12\left(\frac{\omega_{ce0}}{\omega_0}\right)^4\left(\frac{\omega_{pe0}}{\omega_0}\right)^2 + 2\left(\frac{\omega_{pe0}}{\omega_0}\right)^6\left(\frac{\omega_{ce0}}{\omega_0}\right)^2 + 13\left(\frac{\omega_{pe0}}{\omega_0}\right)^4\left(\frac{\omega_{ce0}}{\omega_0}\right)^4 + 8\left(\frac{\omega_{pe0}}{\omega_0}\right)^2\left(\frac{\omega_{ce0}}{\omega_0}\right)^6 + 6\left(\frac{\omega_{ce0}}{\omega_0}\right)^4\left(\frac{\omega_{pe0}}{\omega_0}\right)^6 + \left(\frac{\omega_{ce0}}{\omega_0}\right)^6\left(\frac{\omega_{pe0}}{\omega_0}\right)^4 + 4\left(\frac{\omega_{ce0}}{\omega_0}\right)^6\left(\frac{\omega_{pe0}}{\omega_0}\right)^6 + 4\left(\frac{\omega_{ce0}}{\omega_0}\right)^8\left(\frac{\omega_{pe0}}{\omega_0}\right)^4\right]E_y E_y^*, \quad (11)$$

Proposing (1< γ <2) [19]. The relativistic factor γ may be written as follows.

$$\gamma \approx 1 + \alpha E_y E_y^* \quad (12)$$

Thus, by equaling terms of order $E_y E_y^*$ in both Eq.(11) and Eq.(12) we will obtain

$$\alpha = \frac{1}{4}\left(\frac{e}{m_0\omega_0 c}\right)^2\left[1 + 3\left(\frac{\omega_{ce0}}{\omega_0}\right)^2 + 4\left(\frac{\omega_{ce0}}{\omega_0}\right)^2\left(\frac{\omega_{pe0}}{\omega_0}\right)^2 + 2\left(\frac{\omega_{ce}0}{\omega_0}\right)^4 + 5\left(\frac{\omega_{ce0}}{\omega_0}\right)^2\left(\frac{\omega_{pe0}}{\omega_0}\right)^4 + 12\left(\frac{\omega_{ce0}}{\omega_0}\right)^4\left(\frac{\omega_{pe0}}{\omega_0}\right)^2 + 2\left(\frac{\omega_{ce0}}{\omega_0}\right)^2\left(\frac{\omega_{pe0}}{\omega_0}\right)^6 + 13\left(\frac{\omega_{ce0}}{\omega_0}\right)^4\left(\frac{\omega_{pe0}}{\omega_0}\right)^4 + 8\left(\frac{\omega_{ce0}}{\omega_0}\right)^6\left(\frac{\omega_{pe0}}{\omega_0}\right)^2 + 6\left(\frac{\omega_{ce0}}{\omega_0}\right)^4\left(\frac{\omega_{pe0}}{\omega_0}\right)^6 + 8\left(\frac{\omega_{ce0}}{\omega_0}\right)^6\left(\frac{\omega_{pe0}}{\omega_0}\right)^4 + 4\left(\frac{\omega_{ce0}}{\omega_0}\right)^6\left(\frac{\omega_{pe0}}{\omega_0}\right)^6 + 4\left(\frac{\omega_{ce0}}{\omega_0}\right)^8\left(\frac{\omega_{pe0}}{\omega_0}\right)^4\right] \quad (13)$$

where α is the relativistic nonlinearity brought on by a rise in electron mass, which oscillates in a highly strong intense laser beam and eventually becomes zero at the non-relativistic scope ($\gamma = 1$). By Substituting the value of γ from Eq. (12) into Eq. (9), one obtains

$$\varepsilon_r = 1 - \frac{\frac{\omega_{pe0}^2}{\omega_0^2}\left(1 - \frac{\omega_{pe0}^2}{\omega_0^2}\right)}{\left(1 - \omega_{ce0}^2 - \omega_{pe0}^2\right)} + \left[\frac{\frac{\omega_{pe0}^2}{\omega_0^2}\left(\left(1 - \frac{\omega_{pe0}^2}{\omega_0^2}\right)^2 + \frac{\omega_{ce0}^2}{\omega_0^2}\right)}{\left(1 - \omega_{ce0}^2 - \omega_{pe0}^2\right)^2}\right]\alpha E_y E_y^* \quad (14)$$

The equation above can be expressed as ($\varepsilon_r = \varepsilon_0 + \varepsilon_2 E_y E_y^*$), which refers to the non-relativistic and relativistic components of dielectric constant.

Where:

$$\varepsilon_0 = 1 - \frac{\frac{\omega_{pe0}^2}{\omega_0^2}\left(1 - \frac{\omega_{pe0}^2}{\omega_0^2}\right)}{\left(1 - \omega_{ce0}^2 - \omega_{pe0}^2\right)} \quad (15)$$

And

$$\varepsilon_2 = \left[\frac{\frac{\omega_{pe0}^2}{\omega_0^2}\left(\left(1-\frac{\omega_{pe0}^2}{\omega_0^2}\right)^2 + \frac{\omega_{ce0}^2}{\omega_0^2}\right)}{\left(1-\omega_{ce0}^2-\omega_{pe0}^2\right)^2} \right]\alpha \quad (16)$$

3. Relativistic self-focusing of Hollow Gaussian Laser Beam

The following wave equation can describe the Hollow Gaussian laser beam propagation inside plasma [21]:

$$\nabla^2\vec{E}_0 - k_0\left[1-\frac{1}{k_0}\nabla^2\ln\varepsilon_{eff}\right]\vec{E}_0 = 0 \quad (17)$$

The term $\left|\frac{1}{k_0}\nabla^2\ln\varepsilon_{eff}\right| < 1$ (18)

There in Eq.(17) can be neglected the term $\frac{1}{k_0}\nabla^2\ln\varepsilon_{eff}$. So can be rewriting Eq.(17) as following [21]:

$$\nabla^2\vec{E}_0 - k_0\vec{E}_0 = 0 \quad (19)$$

Where $k_0 = \frac{\omega_0}{c}\varepsilon_{eff}^{1/2}$ can be rewriting Eq.(19) as following :

$$\nabla^2\vec{E}_0 - \frac{\omega_0^2}{c^2}\varepsilon_{eff}\vec{E}_0 = 0 \quad (20)$$

Where in cylindrical coordinate system $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\phi^2} + \frac{\partial^2}{\partial x^2}$ but $\vec{E}_0$ in this case independent on ϕ there the term $\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}$ =0 . So can be rewriting Eq.(20) as follows[19]:

$$\frac{\partial^2\vec{E}_0}{\partial r^2} + \frac{1}{r}\frac{\partial\vec{E}_0}{\partial r} + \frac{\partial^2\vec{E}_0}{\partial x^2} - \frac{\omega_0^2}{c^2}\varepsilon_{eff}\vec{E}_0 = 0 \quad (21)$$

To solve Eq. (21) we introduce [21]

$$E_0 = A_0(r,x)e^{-ik(x)dz} \quad (22)$$

Where $A_0(r,x)$ represent a complex function of space that may be expressed as [23]

$$A_0(r,x) = A_0\exp i\left(k_0 S_+\right) \quad (23)$$

S is the phase of the beam laser and A_0 is a real function.

Putting Eq. (23) and Eq.(22) in Eq. (21), then separating real and imaginary parts we will get

$$2\frac{\partial S_+}{\partial x} + \left(\frac{\partial S_+}{\partial r}\right)^2 = \frac{\varepsilon_2}{\varepsilon_0}A_0^2 + \frac{1}{k_0^2 A_0}\left[\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right] \quad (24)$$

$$\frac{\partial A_0^2}{\partial r}\frac{\partial S_+}{\partial r} + A_0^2\left[\frac{1}{r}\frac{\partial S_+}{\partial r} + \frac{\partial^2 S_+}{\partial r^2}\right] + \frac{\partial A_0^2}{\partial x} = 0 \quad (25)$$

Where Eq.(24) represents the real part, and Eq.(25) represents the imaginary part. Given an initial beam radius of r_0, the initial hollow Gaussian laser beam for (x>0) is given by

$$A_0^2 = \frac{E_{00}^2}{2^{2n} f_0}\left(\frac{r}{r_0 f_0}\right)^{4n} e^{-\left(\frac{r}{r_0 f_0}\right)^2} \quad (26)$$

Where f_0(x) refers to Initial Spot Size of laser beam normalization. Now, one can apply the following equations to determine the location of the maximum intensity in (HGLB) [20]:

$$\eta=\left[\left(\frac{r}{r_0 f_0}\right)-\sqrt{2n}\right] \qquad (27)$$

$$\frac{\partial}{\partial r}=\frac{1}{r_0 f_0}\frac{\partial}{\partial \eta} \qquad (28)$$

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial x}-\frac{\sqrt{2n}+\eta}{f_0}\frac{df_0}{dx}\frac{\partial}{\partial \eta} \qquad (29)$$

by Using Eq.(27) , Eq.(28) and Eq.(29) , can be rewritten Eq.(24)(real part) and Eq.(25)(imaginary part)as follows:

$$2\frac{\partial S_+}{\partial x}+\frac{2\left(\sqrt{2n}+\eta\right)}{f_0}\frac{df_0}{dx}\frac{\partial S_+}{\partial \eta}+\frac{1}{r_0^2 f_0^2}\left(\frac{\partial S_+}{\partial \eta}\right)^2=\frac{\varepsilon_2}{\varepsilon_0}A_0^2+\frac{1}{k_0^2 A_0 r_0^2 f_0^2}\left[\frac{\partial^2 A_0}{\partial \eta^2}+\frac{1}{\left(\sqrt{2n}+\eta\right)}\frac{\partial A_0}{\partial \eta}\right] \qquad (30)$$

$$\frac{1}{r_0^2 f_0^2}\frac{\partial A_0^2}{\partial \eta}\frac{\partial S_+}{\partial \eta}+A_0^2\left[\frac{1}{r_0^2 f_0^2}\frac{\partial^2 S_+}{\partial \eta^2}+\frac{1}{r_0^2 f_0^2\left(\sqrt{2n}+\eta\right)}\frac{\partial S_+}{\partial \eta}\right]+\frac{\partial A_0^2}{\partial x}-\frac{\left(\sqrt{2n}+\eta\right)}{f_0}\frac{df_0}{dz}\frac{\partial A_0^2}{\partial \eta}=0 \qquad (31)$$

We can express the phase of the laser beam as follows.

$$S_+=\frac{\left(\sqrt{2n}+\eta\right)^2}{2}\beta(x) \qquad (32)$$

Now, for magnetized plasma (Transverse magnetized field), And using Eq. (32), The Eq. (31) can be driven to produce as follows:

$$\beta(x)=r_0^2 f\,\frac{df}{dx} \qquad (33)$$

So Eq.(32) can be written as follows.

$$S_+=\frac{r_0^2 f\,\left(\sqrt{2n}+\eta\right)^2}{2}\frac{df}{dx} \qquad (34)$$

Now through the derivation of Eq.(34) with respect to η and x and use Eq.(26),then substituting it into Eq.(30) we get the following equation:

$$\left(\sqrt{2n}+\eta\right)^2 r_0^2 f_0\frac{d^2 f_0}{dx^2}=\frac{\varepsilon_2}{\varepsilon_0}\frac{E_{00}^2}{2^{2n} f_0}\left(\sqrt{2n}+\eta\right)^{4n}e^{-\left(\sqrt{2n}+\eta\right)^2}+\frac{1}{k_0^2 r_0^2 f_0^2}\left[\frac{4n^2-4n}{\left(\sqrt{2n}+\eta\right)^2}-2+\left(\sqrt{2n}+\eta\right)^2\right] \qquad (35)$$

Where:

$$\left(\sqrt{2n}+\eta\right)^{4n}e^{-\left(\sqrt{2n}+\eta\right)^2}=\left(2n\right)^{2n}e^{-2n}\left(1+(4n-1)\eta^2-8n\eta^2+(4n-1)\eta^2+4n(4n-1)\eta^3+2\sqrt{2n}(4n-1)\eta^3+(4n-1)^2\eta^4+\ldots\right) \qquad (36)$$

By taking the terms which related with η^2, So Eq.(35) becomes as follows.

$$\frac{d^2 f_0}{dx^2}=\frac{1}{k_0^2 r_0^4 f_0^3}+\frac{\varepsilon_2}{\varepsilon_0}\frac{E_{00}^2}{r_0^2 f_0^2}\left(-2\left(n\right)^{2n}e^{-2n}\right) \qquad (37)$$

The initial conditions are f (x=0)=1 and df/dx=1 (for an initial plane wavefront). In terms of normalization distance of propagation $\zeta=\frac{x}{k_0 r_0^2}$ and $x=k_0 r_0^2\zeta$, to make Eq. (37) more suitable for programming it using Euler's method and using it in MATLAB, it can be rewritten as follows.

$$\frac{d^2 f_0}{d\zeta^2}=\frac{1}{f_0^3}-2\frac{k_0^2 r_0^2}{f_0^2}\left(\frac{\varepsilon_2}{\varepsilon_0}\right)E_{00}^2\left(\left(n\right)^{2n}e^{-2n}\right) \qquad (38)$$

Where $R_d=k_0 r_0^2$ represents diffraction length and r_0 is the initial radius beam.

$$\frac{d^2 f_0}{d\zeta^2}=\frac{1}{f_0^3}-2\frac{R_d^2}{f_0^2 r_0^2}\left(\frac{\varepsilon_2}{\varepsilon_0}\right)E_{00}^2\left(\left(n\right)^{2n}e^{-2n}\right) \qquad (39)$$

The rivalry between the diffraction and self-focusing factors (first and second terms on the right-hand side of Eq.(39), respectively) results in the Initial Spot Size variation of the laser beam profile in the Paraxial situation, which is represented by Eq.(39). the beamwidth parameter f_0 will vary along the normalized propagation distance ζ periodically

4. Relativistic self-focusing of Hollow Gaussian Laser Beam

This work predates the nonlinear interaction between a carbon dioxide (CO_2) pulsed laser's Hollow Gaussian mode and magnetized plasma. The following set of empirically obtained parameters has been used to solve the final Eq. (39) numerically:

1. for carbon dioxide (CO_2) pulsed laser with wavelength (λ =10.6*10^{-6} µm), frequency (f=2.83*10^{13} Hz), and angular frequency (ω_0 =1.778*10^{14} rad/sec) [22].
2. Magnetic field B= 50530 G, which are corresponding to ω_{ce0} = 0.005 ω_0 [22].
3. the normalized vector potential a_0 =0.7 which are corresponding to the initial laser beam intensities are I=6.3981*10^{20} w/cm^2 where $a_0 = 0.85*10^{-9}\sqrt{I\,(W/cm^2)*\lambda(\mu m)}$ [24].
4. The plasma densities n_0= 5.1085*10^{-18} cm^{-3} which are corresponded to the plasma frequencies ω_{pe0} = 0.99 ω_0 [22].
5. The Hollow Gaussian Laser Beam's (HGLB) order n=2. The Initial Spot Size r_0 = (0.0005, 0.0007, 0.0009, 0.0015, 0.0016, 0.0017, 0.0020, 0.0024, 0.0028) cm.
6. If and only if the laser beam's initial incident power is higher than the critical power, the hollow Gaussian laser beam will exhibit relativistic self-focusing. $\left(P_c \approx 17*10^9*\left(\omega_0/\omega_{pe0}\right)^2\right)$ [25].

- Figure 1 shows the effect of increasing the Initial Spot Size on the self-focusing of (HGLB). the self-focusing limit of (HGLB) in Eq. (39) does not overcome the diffraction limit when the Initial Spot Size is very small, about 0.0005 cm. But when the Initial Spot Size is greater than 0.0005 cm, the self-focusing process of (HGLB) begins to occur.

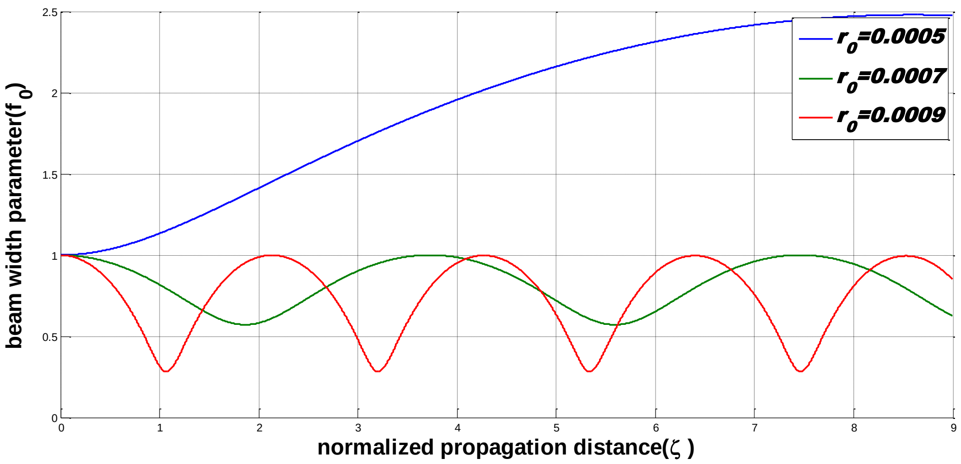

Fig.1 Variation of beam width parameter f_0 with a normalized distance of propagation($\zeta = \frac{x}{k_0 r_0^2}$) for the Hollow Gaussian Laser Beam order (n=2), and different values of Initial Spot Size r_0 = (0.0005, 0.0007, 0.0009) cm**.**

- Figure 2 shows the ability of (HGLB) to self-focusing increases with increasing spot size. As a result of taking advantage of the increase of the order of (HGLB).

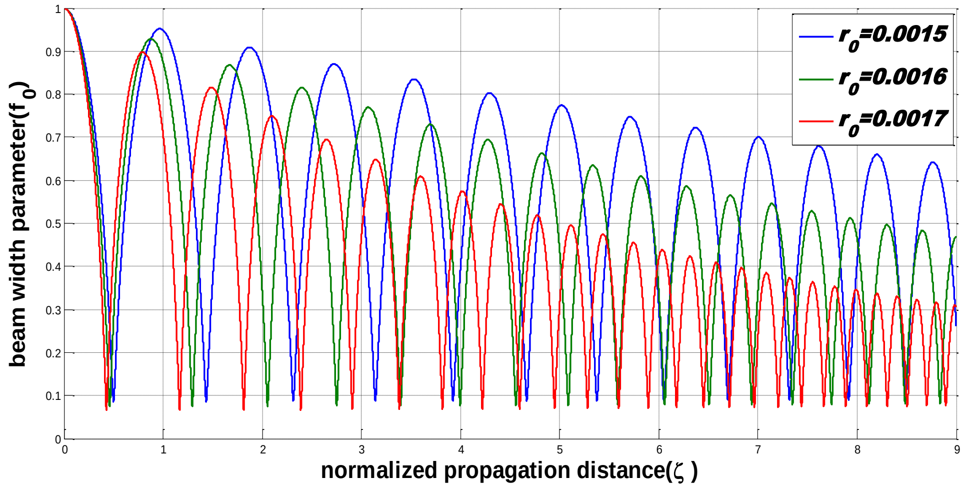

Fig.2 Variation of beam width parameter f_0 with a normalized distance of propagation($\zeta = \frac{x}{k_0 r_0^2}$) for the Hollow Gaussian Laser Beam order (n=2), and different values of Initial Spot Size r_0 = (0.0015, 0.0016, 0.0017) cm.

- Figure 3 shows ability of (HGLB) to self-focusing in the plasma due to the increase in the spot size, because this leads to an increased speed of oscillation of the plasma electrons, enhancing the Hollow Gaussian laser beam ability to focus by raising their relative mass. The figure 3 shows that the self-focusing process of (HGLB) will stabilize at a certain value, and the self-focusing process will be very strong.

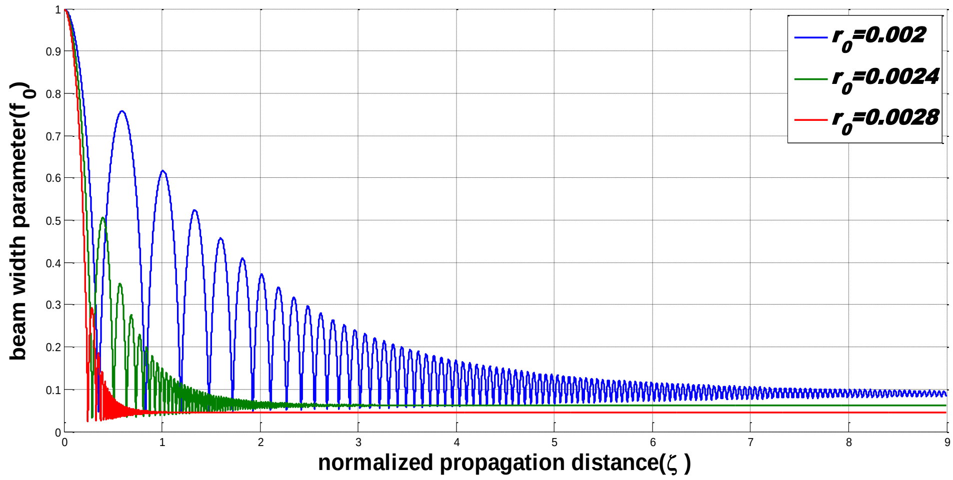

Fig.3 Variation of beam width parameter f_0 with a normalized distance of propagation($\zeta = \frac{x}{k_0 r_0^2}$) for the Hollow Gaussian Laser Beam order (n=2), and different values of Initial Spot Size r_0 = (0.002, 0.0024, 0.0028) cm.

5.Conclusions

In this study, if the second term in Eq. (39), the self-focusing term, triumphs over the first term, the diffraction term, self-focusing of (HGLB) can occur. The self-focusing of (HGLB) of order (n =2) starts when the initial spot size is greater than 0.0005 cm (see Fig.1). The self-focusing of (HGLB) increases with increasing spot size (see Fig.2). the self-focusing process of (HGLB) will stabilize at a certain value, and the self-focusing process will be very strong.

Reference

[1] N. Opačak *et al.*, "Nozaki–Bekki solitons in semiconductor lasers," *Nature*, vol. 625, no. 7996, pp. 685–690, Jan. 2024, doi: 10.1038/s41586-023-06915-7.

[2] S. Chen *et al.*, "Recent progress in Fano-resonant terahertz metasurface and its application," *Discover Applied Sciences*, vol. 6, no. 1, p. 6, Jan. 2024, doi: 10.1007/s42452-024-05661-3.

[3] B. Liu *et al.*, "Terahertz ultrasensitive biosensor based on wide-area and intense light-matter interaction supported by QBIC," *Chemical Engineering Journal*, vol. 462, p. 142347, Apr. 2023, doi: 10.1016/j.cej.2023.142347.

[4] A. A. Hassan, H. F. Hasheem, and M. F. AL-Ruwashdi, "Studying the Absorption and Fluorescence Parameters of Ho: YAP Laser Using the Judd-Oflet Model," *Indian Journal Of Science And Technology*, vol. 17, no. 5, pp. 418–425, Jan. 2024, doi: 10.17485/IJST/v17i5.2866.

[5] Ali J.Yousif, Kadhim J. Kadhim, and Assel M.Abd Al-Mjed, "DepositionFe2O3 Doping Barium Ferrites Thin Films on Glass without Annealingand Study their Microwave Properties ," Mustansiriyah journal of pure and applied sciences, vol. 1, pp. 155–163, 2023.

[6] A. A. Hassan, Sahib N Abdul Wahid, and Haider Y Hamood, "Numerical modeling of passively Q-switched Nd: YAG lasers with Cr+4: YAG as a saturable absorber," *Journal of Xidian University*, vol. 15, no. 2, Feb. 2021, doi: 10.37896/jxu15.2/005.

[7] H. K. Malik and L. Devi, "Relativistic self focusing and frequency shift of super-Gaussian laser beam in plasma," *Results in Physics*, vol. 17, p. 103070, Jun. 2020, doi: 10.1016/j.rinp.2020.103070.

[8] K. Walia, "Self-focusing of q-Gaussian beam in unmagnetized plasma and its impact on second harmonic generation," *Optik*, vol. 277, p. 170681, Apr. 2023, doi: 10.1016/j.ijleo.2023.170681.

[9] C. Schwab *et al.*, "Laser techniques applied to isotope separation of uranium," *Progress in Nuclear Energy*, vol. 33, no. 1–2, pp. 217–264, Jan. 1998, doi: 10.1016/S0149-1970(97)00100-5.

[10] C. Schwab *et al.*, "Laser techniques applied to isotope separation of uranium," *Progress in Nuclear Energy*, vol. 33, no. 1–2, pp. 217–264, Jan. 1998, doi: 10.1016/S0149-1970(97)00100-5.

[11] M. B. Hassan, I. J. Abd-Ali, W. H. Mahdi, and A. H. O. Alkhayatt, "Terahertz wave excitation by nonlinear coupling of intense laser field with magnetized plasma," *Optical and Quantum Electronics*, vol. 55, no. 3, p. 275, Mar. 2023, doi: 10.1007/s11082-023-04557-7

[12] A. M. Hamed and T. A. Al-Saeed, "Monte Carlo simulation of Hermite Gaussian laser beam in turbid medium," *Indian Journal of Physics*, vol. 98, no. 1, pp. 357–364, Jan. 2024, doi: 10.1007/s12648-023-02795-9.

[13] A. Kumar, S. P. Mishra, A. Kumar, and A. Varma, "Electron Bernstein wave aided cosh-Gaussian laser beam absorption in plasma," *Optik*, vol. 273, p. 170436, Feb. 2023, doi: 10.1016/j.ijleo.2022.170436.

[14] S. Kumar, N. Kant, and V. Thakur, "THz generation by self-focused Gaussian laser beam in the array of anharmonic VA-CNTs," *Optical and Quantum Electronics*, vol. 55, no. 3, p. 281, Mar. 2023, doi: 10.1007/s11082-023-04562-w.

[15] N. Gupta *et al.*, "Self-focusing, self-trapping and self-phase modulation of elliptical q-Gaussian laser beams in collisionless plasma," *Journal of Optics*, vol. 53, no. 1, pp. 181–196, Feb. 2024, doi: 10.1007/s12596-022-01091-9.

[16] G. Purohit, R. K. Sharma, and P. Kothiyal, "Relativistic-ponderomotive effect on the self-focusing of a high-intensity cosh-Gaussian laser beam in plasma: higher-order paraxial regime," *Journal of Optics*, vol. 52, no. 3, pp. 1162–1170, Sep. 2023, doi: 10.1007/s12596-022-00931-y.

[17] P. P. Nikam, P. P. Patil, V. S. Pawar, M. v. Takale, and S. D. Patil, "Combined effect of relativistic and ponderomotive nonlinearities on self-focusing of asymmetric finite Airy–Gaussian laser beams in plasma," *Journal of Optics*, Dec. 2023, doi: 10.1007/s12596-023-01522-1.

[18] N. Sepehri Javan and Zh. Nasirzadeh, "Self-focusing of circularly polarized laser pulse in the hot magnetized plasma in the quasi-neutral limit," *Physics of Plasmas*, vol. 19, no. 11, Nov. 2012, doi: 10.1063/1.4766753.

[19] K. I. Hassoon, A. K. Sharma, and R. A. Khamis, "Relativistic laser self-focusing in a plasma with transverse magnetic field," *Physica Scripta*, vol. 81, no. 2, p. 025505, Feb. 2010, doi:

10.1088/0031-8949/81/02/025505.
[20] Abdulkareem Ali Nasser and Munther B. Hassan, “The Self-focusing of High Power Hollow Gaussian Laser Beam in Magnetized Plasma by Operating the Relativistic Nonlinearity,” *Journal of Survey in Fisheries Sciences*, vol. 10, no. 1, pp. 1933–1939, 2023.
[21] W. J. Firth, “Self-focusing of laser beams in dielectrics, plasmas and semiconductors,” *Optics & Laser Technology*, vol. 7, no. 3, p. 141, Jan. 1975, doi: 10.1016/0030-3992(75)90076-6.
[22] A. A. Hassan, A. K. H. Dagher, and M. B. Hassan, “The self-focusing of hollow gaussian laser beam in magnetized plasma with transverse magnetic field by operating relativistic nonlinearity,” *Journal of Optics*, May 2024, doi: 10.1007/s12596-024-01842-w.
[23] S. A. Akhmanov, A. P. Sukhorukov, and R. v Khokhlov, “Self-focusing and diffraction of light in a nonlinear medium,” *Soviet Physics Uspekhi*, vol. 10, no. 5, pp. 609–636, May 1968, doi: 10.1070/PU1968v010n05ABEH005849.
[24] D. Umstadter, “Relativistic laser plasma interactions,” *Journal of Physics D: Applied Physics*, vol. 36, no. 8, pp. R151–R165, Apr. 2003, doi: 10.1088/0022-3727/36/8/202.
[25] P. Sprangle, C.-M. Tang, and E. Esarey, “Relativistic Self-Focusing of Short-Pulse Radiation Beams in Plasmas,” *IEEE Transactions on Plasma Science*, vol. 15, no. 2, pp. 145–153, 1987, doi: 10.1109/TPS.1987.4316677.