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RESEARCH ARTICLE - COMPUTER SCIENCE

Development and Evaluation of an Innovative Three-Dimensional Hyperchaotic map

Walaa Hadi Abdulnabi[1*], Sadiq A. Mehdi [2], Sawsen Abdulhadi Mahmood [3]

[1] Department of Computer Science, Collage of Education, Mustansiriyah University, 10052, Baghdad, Iraq, walaahadi@uomustansiriyah.edu.iq

[2] Department of Computer Science, Collage of Education, Mustansiriyah University, 10052, Baghdad, Iraq

[3] Department of Computer Science, Collage of Education, Mustansiriyah University, 10052, Baghdad, Iraq

[*] Corresponding author E-mail: walaahadi@uomustansiriyah.edu.iq

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Abstract

In our work, we are designing the new system which is 3D hyper-chaotic and subsequently examined the most critical characteristics of this proposed system with a dynamic behavior by using Mathematica software that performs numerical simulations of these dynamic behaviors. The findings indicated that the suggested system has three-dimensional with generates hyper-chaotic attractors, as it has two positive exponents of Lyapunov in (LE1= 1.14247, LE2= 0.01498) After applying the Lyapunov exponent equations, which makes it hyper-chaotic, and that it is unstable and possesses the butterfly effect, as its wave form is non-periodic and therefore has non-periodic properties and is dissipative. It has a Kaplan-York dimension as (2.07643). Also the new system is sensitive to any, even very slight, change in initial conditions, is unpredictable, and is complex, and thus possesses the basic dynamical properties of chaotic systems. Due to its high randomness and sensitivity to initial circumstances and conditions, we suggest using our new system in information security and building an encryption system.

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Keywords: chaotic, Lyapunov exponents, hyper, three-dimensional, unstable, Kaplan-York dimension, sensitivity for initial conditions.

1. Introduction

The world's first dynamical system to exhibit single three-dimensional chaotic attractors and have a remarkably complex and nonlinear chaotic dynamical behavior is the Lorenz system, which was found in 1963 by Lorenz, an American physicist, as he was studying weather prediction [1]. This discovery of the Lorenz system gave rise to the study of chaos and the discovery of the vast array of uses for chaos (For example, in fields like biology, robotics, population dynamics, engineering, computer science, and mathematics) which made the study of chaos production essential[2]. It led to the interest of the scientific community in chaotic systems and their widespread use in many fields - such as physics, biology, complex networks, economics, and more [3]. chaotic systems characterized by its random nature, susceptibility to beginning conditions and parameters, perplexity, periodic patterns [4]. In this work, we presented a new hyperchaotic system that operates in three dimensions. Compared with low-dimensional chaos, our chaotic system exhibits chaotic behavior over a wide range of parameter values, have a large key space, more complexity, and high randomness.

2. Related Work

We will include here some previous studies of three-dimensional chaotic systems:

In 2019[5], Shiyu Guo et al, presented a brand-new, hyperchaotic, 3D autonomous system, In System (1), this nonlinear chaotic system is expressed:

$$\begin{cases} x1(n) = ax1(n-1) + bx2(n-1) + cx3(n-1) + dx1(n-1)\,x2(n-1) + ex1(n-1)\,x3(n-1) + fx2(n-1)\,x3(n-1) \\ x2(n) = x1(n-1) \\ x3(n) = x2(n-1) \end{cases} \quad (1)$$

The parameters value as follows: a = -0.54, b = -0.25, c = 0.79, d = -1.79%,
e= -1.69%, f = -1.78%. Iteration can be used to enter the chaotic state where the initial values: x1(0)=0.63, x2(0)=0.81, and x3(0)=-0.75[5] .

In 2020[6], Dawood Shah et al, introduced the three-dimensional chaotic map and went into great detail on their qualities, which are listed below in (2):

$$\begin{cases} \mathcal{X}i+1 = yi + \alpha \sin(\mathcal{X}i) + \gamma \cos(zi) \\ yi+1 = \mathcal{X}I + \sin(\mathcal{X}i)\cos(yi) + \tan(zi) \\ zi+1 = \mathcal{X}i\sin(i) + yi\cos(i) + \beta\tan{-1}(zi) - \delta \end{cases} \quad (2)$$

For every $\alpha \in [5,\infty)$ in the preceding chaotic equation system, $\beta \in [-10, 10]$ and $(\gamma.\delta) \leq [-1,1]$. And in this chaotic orbited of the system of equations x, y, z for the first 50,000 iterations with the initial condition (x0. y0. z0) = (.0705,00001, 0038) [6].

In 2021[7], Ahmed Jasim Khader and Sadiq Abdul Aziz Mehdi suggested two novel 3D chaotic systems where the first 3D chaotic system is expressed in (3):

$$\begin{cases} \mathcal{X} = -\alpha y - b\mathcal{X} + cyz \\ y = -d\mathcal{X}z - ey - fz\sin(\mathcal{X}) \\ z = -b\,y\sin(y) - g\mathcal{X}y - hz \end{cases} \quad (3)$$

And the second 3D chaotic system is expressed in (4):

$$\begin{cases} \mathcal{X} = \alpha\, y2 - b\mathcal{X} + cyz \\ y = d\mathcal{X}2z + ey - f\,z \\ z = -g\mathcal{X}y - hz \end{cases} \quad (4)$$

When the system parameter values are set at a=25, b=10, c=10.1, d=17, e=7, f=15, g=6, and h=4, along with the initial conditions $x_{(0)}$= -1, $y_{(0)}$=5.15, and $z_{(0)}$=0.7, the first 3D chaotic system (3) displays a chaotic attractor. The three Lyapunov exponents of the nonlinear dynamical system (3) are L1= 5.91224, L2= -4.16653 and L3= -22.7337. As can be observed, the system exhibits chaotic properties because the greatest Lyapunov exponent is positive. L2 and L3 are two negative Lyapunov exponents, whereas L_1 is the single positive exponent. The system is hence chaotic. Figure (1.4) showed phase portrait of the first system (3).

When a=17, b=1.5, c=12, d=30, e=4.5, f=10, g=20, and h=15 are selected as the system parameter values, the 3D chaotic system (4) displays a chaotic attractor. The two Lyapunov exponents of a second nonlinear dynamically system (4) are derived like follows, with the initial conditions being $x_{(0)}$=0.2, $y_{(0)}$=0.5, and z(0)=0.7, L_1 = 1.33576, L2 = -0.00841839, L3 = 13.3269. The system exhibits chaotic characteristics as the greatest Lyapunov exponent is positive. The two negative Lyapunov exponents are L2 and L3, while the single positive Lyapunov exponent is L1. The result is chaos in the system[7].

3. New Three-Dimensional Chaotic System Construction

The main goal is to create a mathematical model represents a new three dimension chaotic system. The new 3-Dimensional autonomous system is obtained as follows:

$$\begin{cases} \frac{dx}{dt} = a\,y^2 - b\,x - c\,y^2 z \\ \frac{dy}{dt} = d\,x^2 z + e\,y - f\,x\,z \\ \frac{dz}{dt} = -g\,x^2 y - h\,z - i\,y^2 \end{cases} \qquad (5)$$

An arithmetic form has nine parameters and state variables that have a positive value, parameters namely $a, b, c, d,$ with $e, f, g, h, i,$ also state variables namely $x, y, z,$ describes a chaotic 3D system. The system displays a chaotic attractor characterized by specific parameter together with initial values which are $a = 12, b = 2.5, c = 7.5, d = 32, e = 4.5, f = 19, g = 24, h = 16, i = 15$ within the initial values such as: $x(0) = 0.3, y(0) = 0.4, z(0) = 0.2$.

4. Dynamic Evaluation of Novel 3D Chaotic System

This part examines the fundamental and intricate dynamic behavior of the new system, which possesses the following fundamental characteristics:

4.1. The point of equilibrium or The Fixed point

refers to a specific value in a system where the variables remain constant across time. The system (5) exhibits a single fixed point E1(0,0,0) under the following conditions:

$$\begin{aligned} 0 &= a\,y^2 - \mathrm{b}\,x - c\,y^2 z \\ 0 &= d\,x^2 \mathrm{z} + e\,y - \mathcal{F}\,x\,z \qquad (6) \\ 0 &= -g\,x^2 y - h\,\mathrm{z} - i\,y^2 \end{aligned}$$

The Jacobian matrix of the system (5) is as follows:

$$\begin{aligned} \mathcal{F}1 &= \frac{dx}{dt} = a\,y^2 - b\,x - c\,y^2 z \\ \mathcal{F}2 &= \frac{dy}{dt} = d\,x^2 z + e\,y - \mathcal{F}\,x\,z \qquad (7) \\ \mathcal{F}3 &= \frac{dz}{dt} = -g\,x^2 y - h\,z - i\,y^2 \end{aligned}$$

$$J = \begin{bmatrix} \frac{\partial \mathcal{F}_1}{\partial x} & \frac{\partial \mathcal{F}_1}{\partial y} & \frac{\partial \mathcal{F}_1}{\partial z} \\ \frac{\partial \mathcal{F}_2}{\partial x} & \frac{\partial \mathcal{F}f_2}{\partial y} & \frac{\partial \mathcal{F}_2}{\partial z} \\ \frac{\partial \mathcal{F}_3}{\partial x} & \frac{\partial \mathcal{F}_3}{\partial y} & \frac{\partial \mathcal{F}_3}{\partial z} \end{bmatrix} \qquad (8)$$

$$= \begin{bmatrix} -b & 2ay - 2cyz & -c\,y^2 \\ 2dxz - \mathcal{F}z & e & dx^2 - \mathcal{F}x \\ -2g\,xy & -g\,x^2 - 2i\,y & -h \end{bmatrix} \quad (9)$$

For the point of equilibrium: $E1\{x = 0, y = 0, z = 0\}$ and system parameters (6), the Jacobian matrix has the following result:

$$J_{1} = \begin{bmatrix} -2.5 & 0 & 0 \\ 0 & 4.5 & 0 \\ 0 & 0 & -16 \end{bmatrix} \quad (10)$$

In order to derive the eigenvalues, consider the case where $[\lambda \boldsymbol{I} - \boldsymbol{J}0]$ is equal to zero. Subsequently, the equilibrium eigenvalues $E1(0,0,0)$ a are obtained in the following manner:

$\boldsymbol{\lambda\_1 = -16, \lambda\_2 = 4.5 \; and \; \lambda\_3 = -2.5}$.

For the equilibrium points 1 , The findings indicate that $\lambda 1 \; and \; \lambda 3$ are actual numbers

that are negative. Therefore, equilibrium points E1 is unstable".

4.2. Dissipativity

This new system of chaotic in (5), it will be expressed in vector notation like (7). The

vector field f's divergence onto R3 is provided by:

$$\nabla . \mathcal{F} = \frac{\partial \mathcal{F}_1}{\partial x} + \frac{\partial \mathcal{F}_2}{\partial y} + \frac{\partial \mathcal{F}_3}{\partial z} \quad (11)$$

It should be noted that the rate of volume change under the influence of flow $\Phi \boldsymbol{t} \; of \; \boldsymbol{\mathcal{F}}$ is denoted by $\boldsymbol{\nabla . \mathcal{F}}$.

Consider an area Ɗ into R3 along with the smooth boundary· When Ɗ $(\boldsymbol{t})$ be an image for Ɗ when it's under the flow of $\mathcal{F}$ at time t, denoted as $\Phi \boldsymbol{t}$. Define $\boldsymbol{V(t)}$ as the volume of D(t). Applying Liouville's theorem [8], we obtain:

$$\frac{dv}{dt} = \int_{D(t)} (\nabla . \mathrm{f}) \, dx \, dy \, dz \quad (12)$$

By applying equation (5), we can deduce equation (11)

$$= (-b + e - h) < 0 \quad (13)$$

because $b, e, h \; are$ a positive constant. Substituting (13) into (12) and simplifying, we get:

$$\frac{dv}{dt} = (-\mathrm{b} + e - h) \int_{\mathrm{D(t)}} dx \, dy \, dz \quad (14)$$

$$= (-\mathrm{b} + \mathrm{e} - h) V(\mathrm{t})$$

$$= \boldsymbol{e}^{(-\boldsymbol{b}+\boldsymbol{e}-\boldsymbol{h})} V(t)$$

By solving the first order linear differential equation (13), we are able to get the special resolution:

$$V(t) = V(0)e^{(-b+e-\hbar)t} \;\; = V(0)e^{-14t} \tag{15}$$

Any volume $V(t)$ must drop exponentially quickly to zero over time, as demonstrated by $Eq.$ (3.11). This means that the dynamical system that (5) describes is a dissipative system.

4.3. The Invariability with the Symmetry

The first new chaotic system that results from converting (x,y,z) to $(x,-y,-z)$ will be symmetric about the z-axis and is invariant[8]·

To illustrate the conclusion, let us:

$$x = x,\ y = -y,\ z = -z\cdot \tag{16}$$

and then we have:

$$\frac{dx}{dt} = \frac{dx}{dt}, -\frac{dy}{dt} = \frac{dy}{dt}\ and -\ \frac{dz}{dt} = \frac{dz}{dt} \tag{17}$$

Based on Eq· (16) with Eq· (17), the outcome is acquired in a following:

$$\frac{dx}{dt} = \ a\,y^2 - b\,x - c\,y^2 z$$

$$-\frac{dy}{dt} = -\,d\,x^2 z - e\,y + f\,x\,z \tag{18}$$

$$-\frac{dz}{dt} = g\,x^2 y + h\,z + i\,y^2$$

$\Rightarrow$

$$\frac{dx}{dt} = \ a\,y^2 - b\,x - c\,y^2 z$$

$$\frac{dy}{dt} = \ d\,x^2 z + e\,y - f\,x\,z \tag{19}$$

$$\frac{dz}{dt} = -g\,x^2 y - h\,z - i\,y^2$$

The system (5) exhibits invariance under direction when the coordinates are changed from $(x,y,z)\ to\ (x,-y,-z)$, which remains valid for all system parameter values. Hence, the system (5) exhibits rotational symmetry around the z-axis, implying that any non-trivial trajectory of the system (5) will necessarily have a corresponding twin trajectory. It is evident that the z-axis remains unchanged when the system flows (5)·

4.4. The Dimensions of Lyapunov with Exponents of Lyapunov.

As per the principles of nonlinear dynamical theory, the Lyapunov exponent is computed like a measure for quantitative for a sensitive dependence for the beginning conditions, which is representing a mean rate for convergence or also divergence between the two trajectories that are adjacent[9]. Furthermore, a following formulas are used to derive the three exponents of Lyapunov of a nonlinear dynamical system (5) and the parameters (6):

LE_1= 1.14247**,** **LE_2=** 0.01498**and** **LE_3=** -15.1438**.**

It is evident for a biggest exponent of Lyapunov was positive, signifying a presence of the properties of chaotic in the system. Given that both LE_1 with LE_2 are positive Lyapunov exponents, it may be concluded for this system is a **hyper-chaotic**. A dimension of fractal Briefly is a property for the chaos which that commonly specified using a dimension of Kaplan-Yorke derived via exponents of Lyapunov. A value of D_{KY} can be represented in[10]:

$$D_{KY} = j + \frac{1}{|L_{j+1}|}\sum_{i=1}^{j} L_i \qquad (20)$$

j represents the highest value for the exponent of Lyapunov that satisfies the conditions of $\sum_{i=1}^{j} LE_i > 0$ with $\sum_{i=1}^{j+1} LE_i < 0$ simultaneously. LEi is arranged in descending order based on the sequence of Lyapunov exponents. The(D_{KY}) symbol is representing a limit of maximum for a dimension into a system information. An exponents of Lyapunov for a system in this study were observed, and it was found that the value of $(j = 2)$. By applying the conditions $LE1 + LE2 > 0$ with $LE1 + LE2 + LE3 < 0$, the dimension of Kaplan-Yorke of this new system of chaotic may be determined:

$$= 2 + \frac{LE_1 + LE_2}{|LE_3|} = \mathbf{2.07643}$$

Consequently, the dimension of Lyapunov for system in (5) has value's non-integer. caused by its fractal character, an initial system displays the orbits of non-periodic, and as well¸ they have neighboring paths diverge. Hence, there exists genuine disorder in this non-linear system.

4.5. The portraits of Phase

The portraits of Phase corresponding into the parameters of (6) are shown in Fig.'s ((1) – (12)). The initial chaotic attractor displays a highly intriguing, intricate, and chaotic dynamical pattern.

The numerical simulation has been done by using a **MATHEMATICA** application. This system, which is not linear, displays intricate and plentiful chaotic dynamics. The 3D unusual attractors are depicted in Figure's. The range of values from (5) to (10) and the peculiar attractors in 2D are depicted in Fig.'s. (7) to (12). As depicted in Fig.'s. (1) to (6), the topology bears a resemblance to a flying butterfly with its wings in motion, hence a term "**Butterfly Effect**".

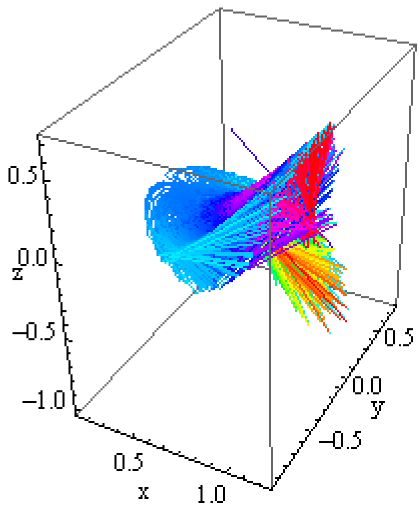

Fig. 1. Portraits of the phase

For a system into (x, y, z).

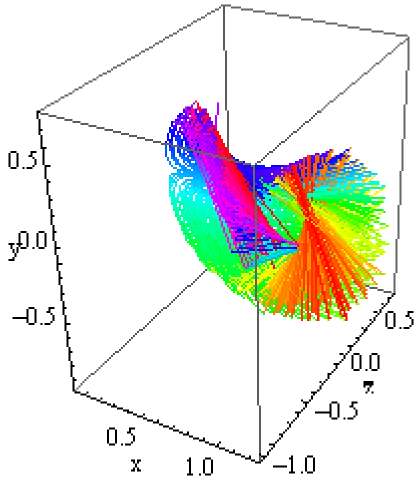

Fig. 2. Portraits of the phase

For a system into (x,z,y).

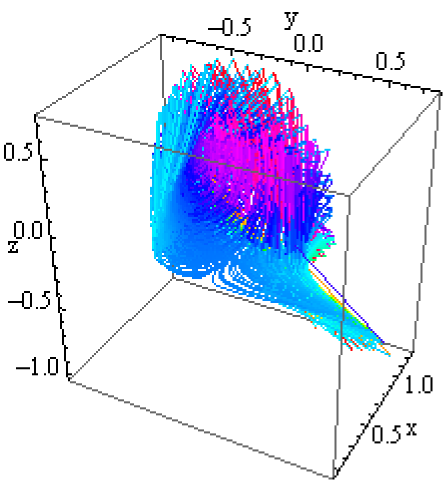

Fig. 3. Portraits of the phase

For a system into (y, x, z).

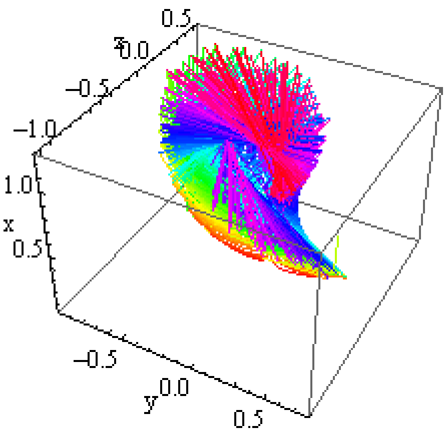

Fig. 4. Portraits of the phase

For a system into (y, z, x).

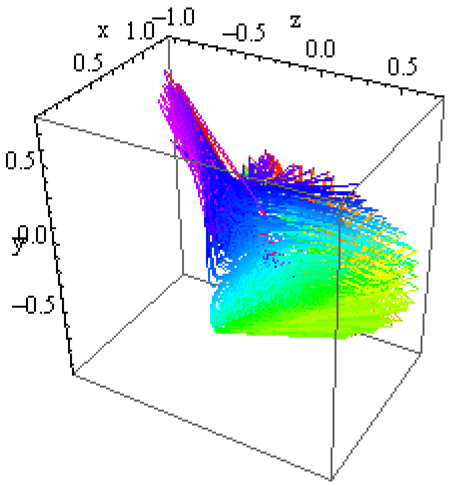

Fig. 5. Portraits of the phase

For a system into (z,x,y).

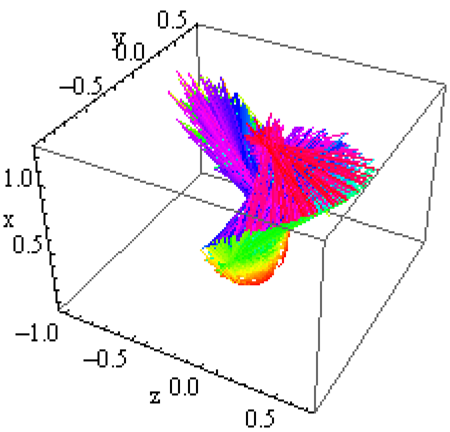

Fig. 6. Portraits of the phase

For a system into (z,y,x).

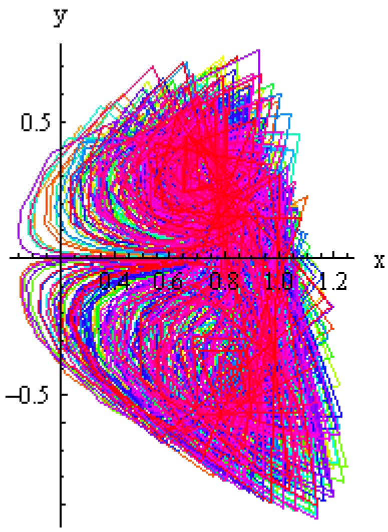

Fig. 7. Portraits of the phase

For a system into (¸y)·

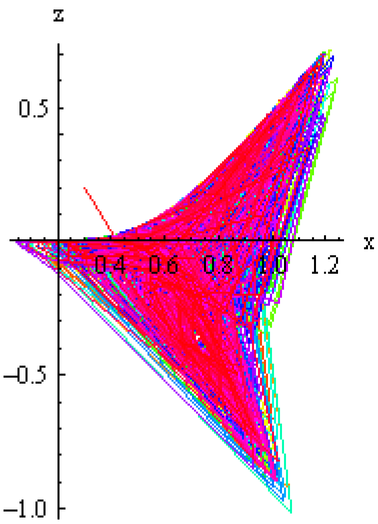

Fig. 8. Portraits of the phase

For a system into (x¸z)·

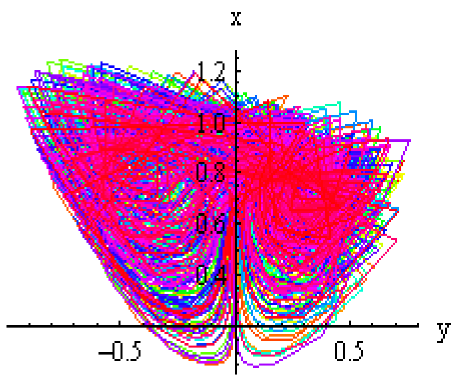

Fig. 9. Portraits of the phase

For a system into (¸x)·

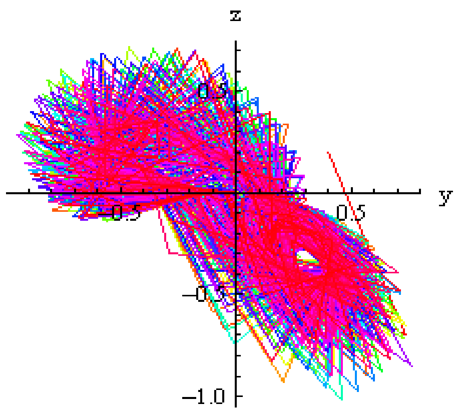

Fig. 10. Portraits of the phase

For a system into (y¸ z)·

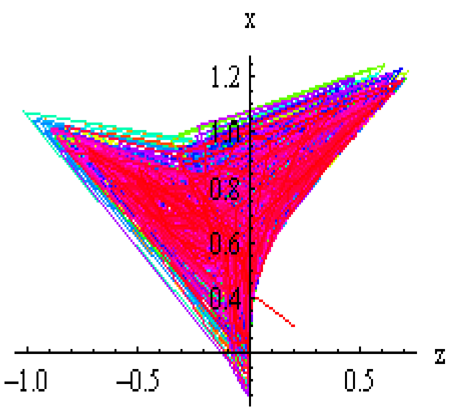

Fig. 11. portraits of the Phase

For a system into (¸ x)

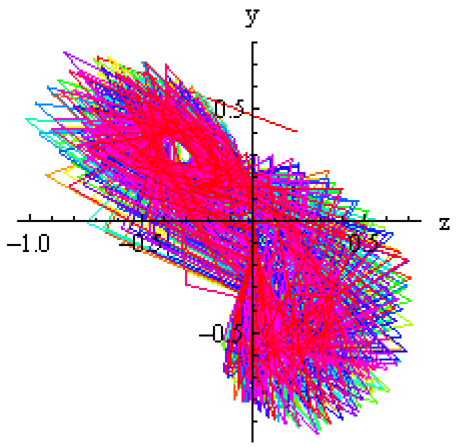

Fig. 12. portraits of the Phase

For a system into (z¸ y)

4.6. Analysis of the waveforms for this system of novel chaotic

It is great recognized for any systems chaotic that a waveform must be non-periodic. For proving the nature of a chaotic for new systems, it is necessary to demonstrate that it exhibits the characteristics of chaos. Fig.'s (13 –15) display the plot of time versus states derived by a **MATHEMATICA** simulation.

The time-domain waveforms for the $(x\cdot(t), y\cdot(t), z\cdot(t))$ variables depicted into Fig.'s (13 – 15). The waveforms for $(x\cdot(t), y\cdot(t), z\cdot(t))$ characterized as **aperiodic**. For distinguish between multiple periodic motion, which may exhibit complex behavior, and chaotic motion, one might observe which a waveform of time domain owns the non-cyclical properties.

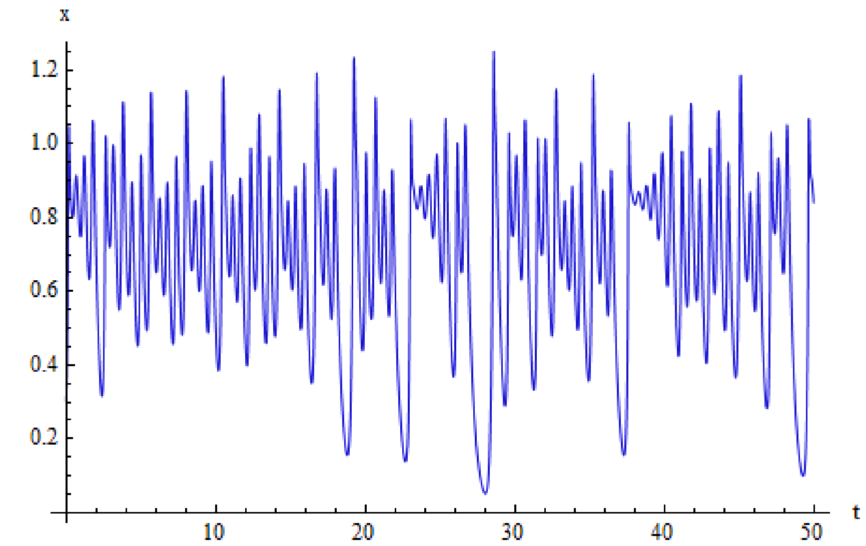

Fig. 13. The first new chaotic system's time versus x

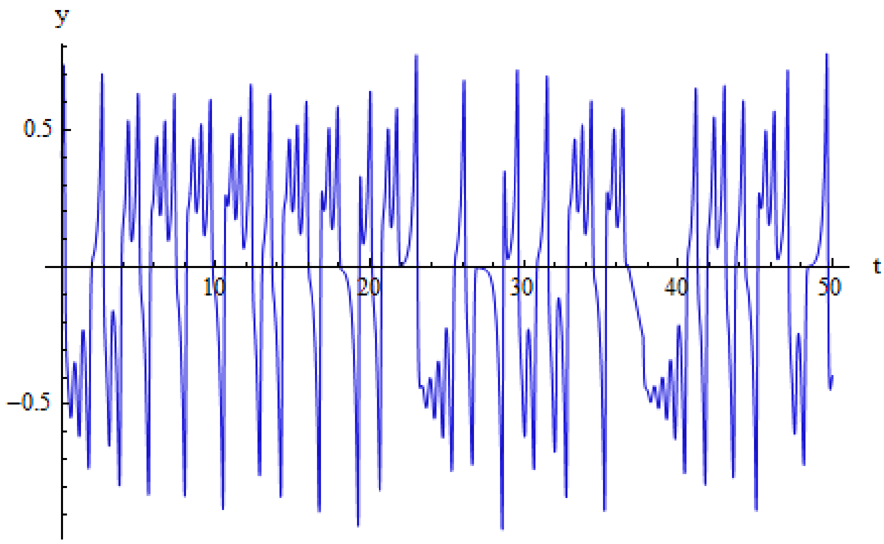

Fig. 14. The first new chaotic system's time versus y

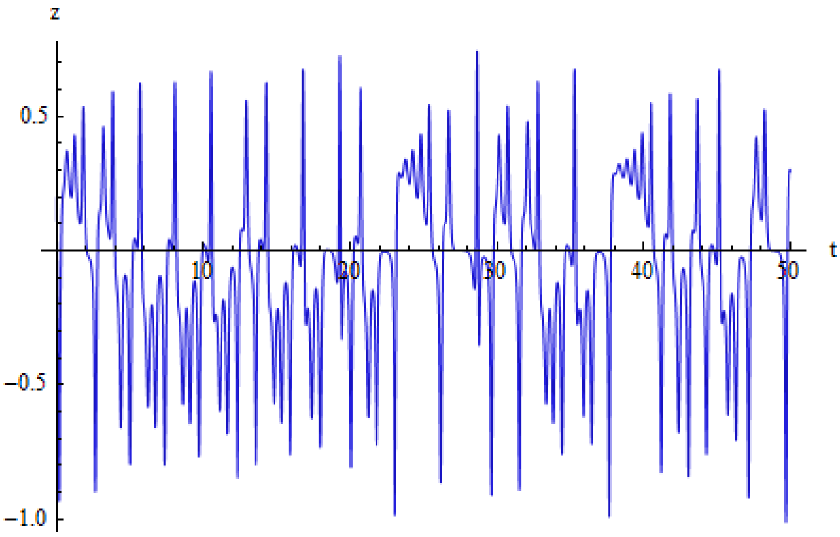

Fig. 15. The first new chaotic system's time versus z

4.7. Initial condition sensitivity

The primary distinguishing property of a chaotic system is its extended unpredictability, resulting from solutions that delicately depend onto initial conditions. Irrespective of their closeness, two separate initial conditions that will inevitably diverge eventually with time. Consequently, a future moment will inevitably occur when an initial condition contains a finite number of digits of precision where it becomes impossible to make accurate predictions around a state of a system.

Fig.'s $(\mathbf{16-18})$ Demonstrates so chaotic trajectory evolution is extremely sensitive into an initial situations. A system is initialized with the following values:

$\boldsymbol{x}(0) = 0.\mathbf{3}, \boldsymbol{y}(0) = \mathbf{0.4}\ , \boldsymbol{z}(\mathbf{0}) = \mathbf{0.2}$ to a line of solid with $\boldsymbol{x}.(0) = \mathbf{0.300000000000001}, \boldsymbol{y}.(0) = \mathbf{0.4}, \boldsymbol{z}.(\mathbf{0}) = \mathbf{0.2}$ for a line of dashed.

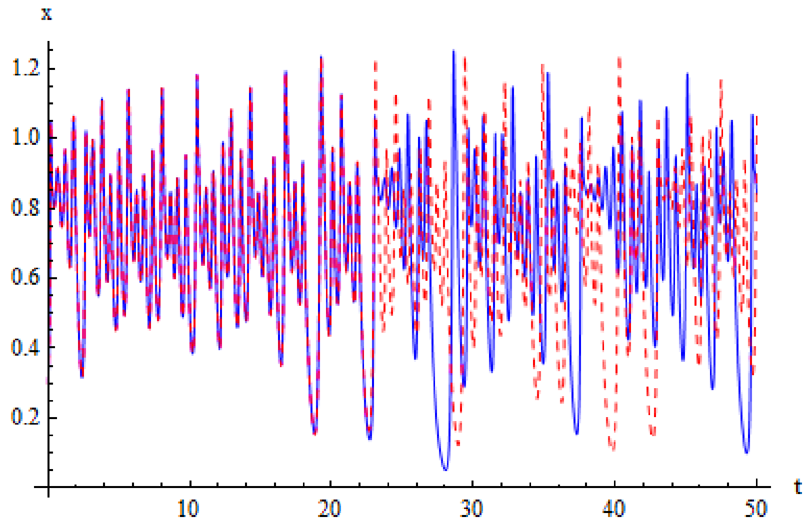

Fig. 16. Evaluation for an initial 3D chaotic system's sensitiveness $\boldsymbol{x}(t)$

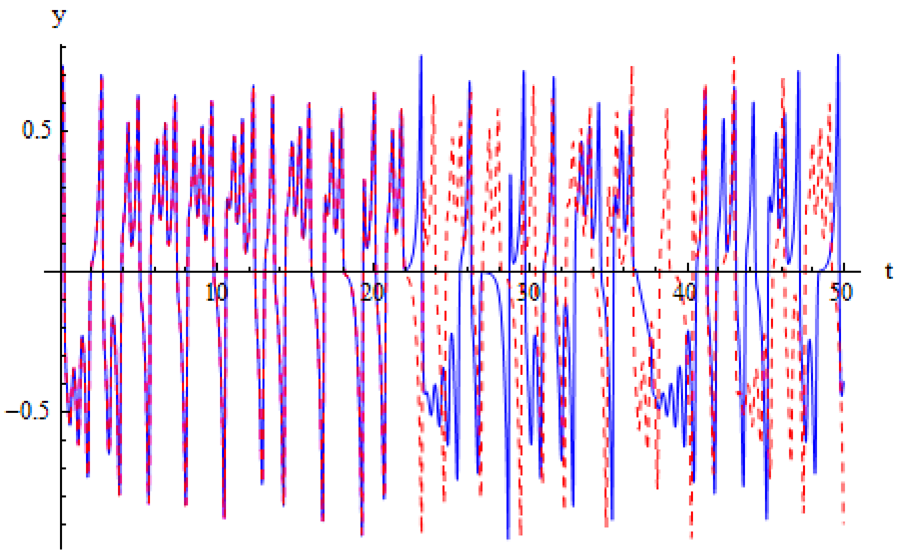

Fig. 17. Evaluation for an initial 3D chaotic system's sensitiveness $\boldsymbol{y}(t)$

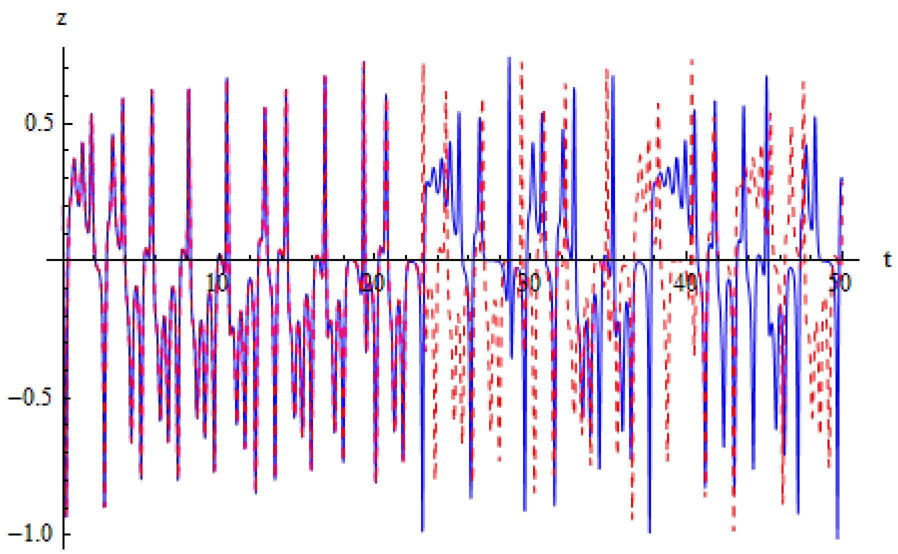

Fig. 18. Evaluation for an initial 3D chaotic system's sensitiveness $\boldsymbol{z}(t)$.

Clearly, a waveform for a system (5) has an aperiodic with exhibits heightened sensitivity to beginning conditions, which we refer to as sensitive dependency on the initial conditions.

4.8. Bifurcation Diagram

The new system (5) is solved numerically through the simulation of Mathematica program and when obtaining the highest values of the variable $\boldsymbol{y}$ and when changing by varying the parameter b, it is possible to discern the region of attention in $\boldsymbol{y}$ which a system changes the conduct and values it exhibits of $\boldsymbol{x}$ begin to bifurcation, especially between b=2.24 and b=2.26 and also at the values between b=2.28 and b=2.30, and the bifurcation property is one of the most crucial attributes of chaotic systems is their attainment in this a new system as shown figure (19).

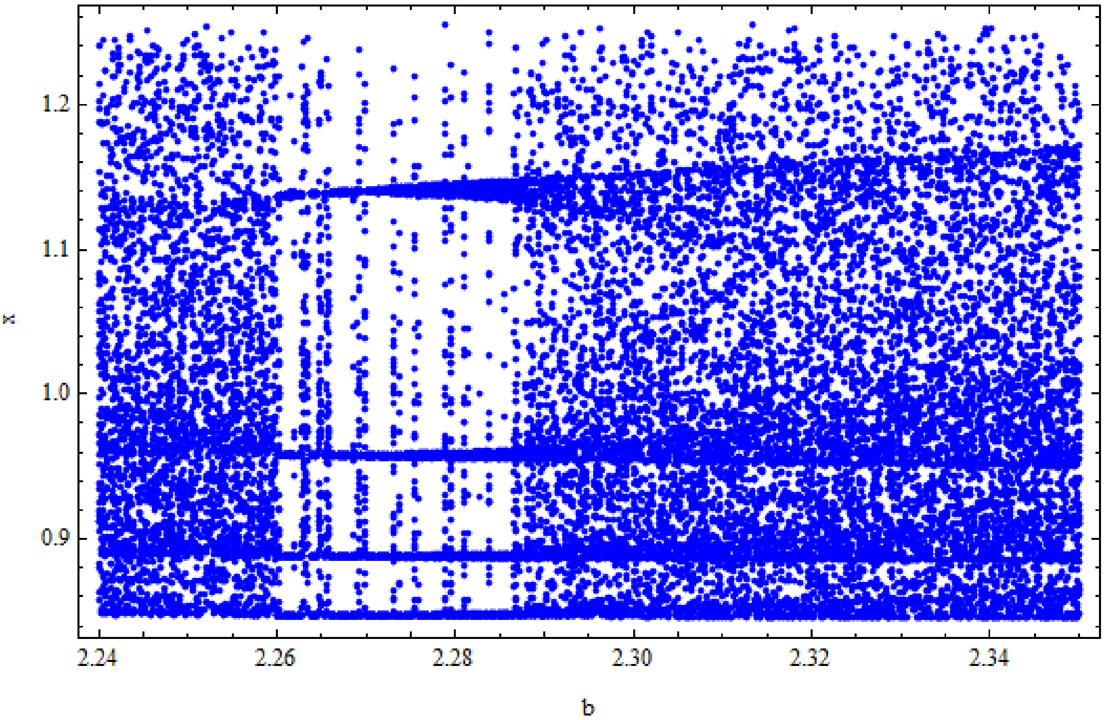

Figure (19): x Bifurcation diagram for increasing k

5. conclusion

This research presents a novel three-dimensional hyperchaotic system. To demonstrate that the new system is chaotic, its fundamental characteristics and dynamic behavior are investigated. With the following parameter selections, the new system exhibits chaotic behavior: a=12, b=2.5, c=7.5, d=32, e=4.5, f=19, g=24, h=16, i=15. (LE1= 1.14247, LE2= 0.01498) are the exponents of Lyapunov of the new system, with initial conditions set at: $\boldsymbol{x(0)} = \mathbf{0.3}, \boldsymbol{y(0)} = \mathbf{0.4}, \boldsymbol{z(0)} = \mathbf{0.2}$. which indicates that the system is hyperchaotic due to the two positive Lyapunov Exponents for the new system, the one unstable equilibrium points, the (2.07643) fractal dimension, the high sensitivity to the starting points, and the generation of complex chaotic attractor. The novel chaotic system could be utilized for information encryption and is appropriate for a wide range of applications.

6.Acknowledgement

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7. Reference

[1] Z. Wu, P. Pan, C. Sun, and B. Zhao, "Plaintext-related dynamic key chaotic image encryption algorithm," *Entropy*, vol. 23, no. 9, 2021, doi: 10.3390/e23091159.

[2] N. A. Saeed, H. A. Saleh, W. A. El-Ganaini, M. Kamel, and M. S. Mohamed, "On a New Three-Dimensional Chaotic System with Adaptive Control and Chaos Synchronization," *Shock and Vibration*, vol. 2023. 2023. doi: 10.1155/2023/1969500.

[3] C. Zou, Q. Zhang, X. Wei, and C. Liu, "Image Encryption Based on Improved Lorenz System," *IEEE Access*, vol. 8. pp. 75728–75740, 2020. doi: 10.1109/ACCESS.2020.2988880.

[4] S. A. Mehdi, K. K. Jabbar, and F. H. Abbood, “Image encryption based on the novel 5D hyper-chaotic system via improved AES algorithm,” *Int. J. Civ. Eng. Technol.*, vol. 9, no. 10, pp. 1841–1855, 2018.

[5] Y. Xie, J. Yu, S. Guo, Q. Ding, and E. Wang, “Image encryption scheme with compressed sensing based on new three-dimensional chaotic system,” *Entropy*, vol. 21, no. 9, 2019, doi: 10.3390/e21090819.

[6] D. Shah, T. Shah, I. Ahamad, M. I. Haider, and I. Khalid, “A three-dimensional chaotic map and their applications to digital audio security,” *Multimedia Tools and Applications*, vol. 80, no. 14. pp. 22251–22273, 2021. doi: 10.1007/s11042-021-10697-3.

[7] A. J. Khader and S. A. A. Mehdi, “A Two Novel Chaotic System for Image Encryption.” Baghdad, Iraq, pp. 30,37, 46, 54, 2021.

[8] R. E. Allen, “Predictions of a Fundamental Statistical Picture,” *Arxiv preprint arXiv:1101.0586*, vol. 0. p. 38, 2011. [Online]. Available: http://arxiv.org/abs/1101.0586

[9] H. R. Shakir, S. A. Mehdi, and A. A. Hattab, “A New Method for Color Image Encryption Using Chaotic System and DNA Encoding,” vol. 1, no. 1. pp. 68–79, 2023.

[10] S. A. Mehdi and D. F. Chalob, “Design And Analysis Of A New Three Dimensional Hyper Chaotic System,” *المجلة العراقية لتكنولوجيا المعلومات*. p. 82, 2018. doi: 10.34279/0923-009-001-009.