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RESEARCH ARTICLE - MATHEMATICS

Coherent system reliability stress-strength model of chen distribution

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Abstract

This research presented reliability formula for multicomponent stress-strength of coherent system, based on Chen Distribution with unknown parameters $\mu, \alpha_i; i = 1,2,3$ and known common parameter β. Three methods discussed for estimating the parameters of Chen distribution that used to estimate the reliability function using Maximum Likelihood, Pitman and Least Square Methods. These estimates are compared via a simulation study using mean square error criteria to large, medium and small samples. The comparison's most important results are that the estimator performs of the maximum likelihood is better in most of the experiments that have been studied.

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***Keywords**:* Coherent System, Stress-Strength Reliability, Chen Distribution

1. Introduction

In reliability research the stress-strength model represents the life-time of a component with random strength X that subject to random stress Y. The classic stress-strength reliability concept involves estimating probability $P(X < Y)$ of component failure, when stress Y exceeds strength X. These model has been extensively investigated under different distributional hypotheses for X and Y [1]. The problem was in finding the reliability of the system in multicomponent stress-strength models because the strengths of various components may differ, and they may be subjected to the same or different stresses. Such situations have been discussed in coherent systems by many authors, J. D. Esary and F. Proscha in 1963, explored some general aspects relating to reliability for coherent systems that components are independent, but may not necessarily have the same reliability [2]. Richard E. Barlow in 1977, generalized the theory of binary coherent systems for multi-state components [3]. Jorge Navarroa et al. in 2005, using Samaniego's signatures, some ordering properties have been extended to coherent systems containing identically distributed components and independent to coherent systems containing (possibly) dependent components [4]. Serkan Eryilmaz in 2008, evaluated the reliability for coherent structures and established a multivariate stress–strength model depending on the conditional ordering of X_is and Y [5]. Debasis Bhattacharya and Soma Roychowdhury in 2013, showed how to derive stress-strength reliability - at least the lower bound - for multicomponent system. A system's stress-strength reliability was expressed to be a function for reliabilities of the stress-strength of its various components [6].

The main aim for this research is to find a mathematical formulation of the reliability R for the multicomponent stress-strength model for coherent system depends on Chen distribution in section 4. Three different methods of estimation (Maximum Likelihood, Pitman and Least Square methods) have been used to obtain estimates of the scale parameters $(\mu, \alpha_1, \alpha_2, \alpha_3)$ for four random variables, and then estimate the reliability parameter in section 5. To compare the performance of different reliability estimates, a simulation study was carried out in Section 6, via nine experiments with values of scale parameter and different sample sizes (15 for small, 30 for medium, and 90 for large). The mean square error criteria are used to do this comparison, and discuss the conclusions in section7.

2. Coherent system

The system's reliability can be described in a number of ways depending on its structure. A binary indicator variable u_i represents the performance of n components in the system and can be defined as: [5], [7]

$$u_i = \begin{cases} 1 & \text{if the } i\text{th component operates} \\ 0 & \text{if the } i\text{th component fails} \end{cases}$$

In the same way, the binary variable $\emptyset$, which represents the system's state as a function of $\mathbf{u} = (u_1, u_2, \dots, u_n)$, can be defined as follows:

$$\emptyset(\mathbf{u}) = \begin{cases} 1 & \text{if the system operates} \\ 0 & \text{if the system fails} \end{cases}$$

This function is known as the system's structure function.

Any system's reliability can be examined using the coherent systems concept. The coherent system can be simply described with binary states u_i and $\emptyset(\mathbf{u})$.

The system is known to be coherent system if

1) The structure function $(\emptyset)$ is non-decreasing in all of its arguments
2) each component is relevant, which means that there is at least one vector **u** for which $\emptyset(1_i, \mathbf{u}) = 1$ and $\emptyset(0_i, \mathbf{u}) = 0$

Where $(0_i, \mathbf{u}) = (u_1, \dots, u_{i-1}, 0, u_{i+1}, \dots, u_n)$

$(1_i, \mathbf{u}) = (u_1, \dots, u_{i-1}, 1, u_{i+1}, \dots, u_n)$

The minimal paths and minimal cuts concept is used to express the structure function of a coherent system. [7],[8]

- **Minimal path**: is the minimal number of components whose operation guarantees the system's operation.

$$\emptyset(\mathbf{u}) = 1 - \prod_{j=1}^{P} \left[1 - \rho_j(\mathbf{u})\right]$$

Where $\rho_j(\mathbf{u})$ is the *j*th minimal path series structure for path A_j.

- **Minimal cut:** is the minimal number of components that, if they failed, the entire system will fail.

$$\emptyset(\mathbf{u}) = \prod_{k=1}^{s} \delta_k(\mathbf{u})$$

Where $\delta_k(\mathbf{u})$ is the *k*th minimal parallel cut structure for cut B_k.

The system's Reliability is given by:

$$R_s = P[\emptyset(\mathbf{X}) = 1]$$

3. Chen distribution

In recent years, some probability distributions have been proposed for modeling the real life data with bathtub shape failure rates. A two-parameter life distribution with bathtub shape or increasing hazard function was proposed by Chen. Let X is a random variable follows Chen distribution then cumulative density function c.d.f is given by: [9]

$$F(x) = 1 - e^{\alpha\left(1-e^{x^{\beta}}\right)} \qquad ; x > 0; \alpha, \beta > 0 \tag{1}$$

Probability density function p.d.f:

$$\mathrm{f}(x) = \alpha\beta x^{\beta-1} e^{x^{\beta}} e^{\alpha\left(1-e^{x^{\beta}}\right)} \qquad ; x > 0; \alpha, \beta > 0 \tag{2}$$

Where α, β are scale and shape parameters respectively.

Since $\mathrm{f}(x)$ is p.d.f and $\int_0^{\infty} \mathrm{f}(x)dx = 1$, then equation (2) can be rewritten as:

$$\int_0^{\infty} \beta x^{\beta-1} e^{x^{\beta}} e^{\alpha\left(1-e^{x^{\beta}}\right)} dx = \frac{1}{\alpha} \tag{3}$$

4. Reliability Formulation

Consider the system in figure.1

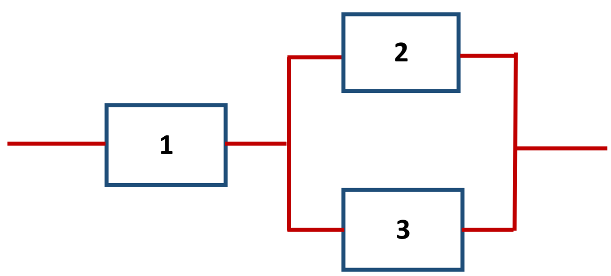

Fig. 1. A series-parallel system

For this system, the minimal cut sets are $\{1\}$ and $\{2, 3\}$. Let the components 1, 2, and 3 have strengths X_1, X_2 and X_3, respectively, and the components are subjected to a common stress variable Y then the system's stress-strength reliability is given by: [6]

$$R = P(X_1 > Y)[P(X_2 > Y) + P(X_3 > Y) - P(X_2 > Y)P(X_3 > Y)] \tag{4}$$

Let Y be a stress random variable having cumulative density functions $G_Y(y)$ following $Y \sim CD(\mu, \beta)$, and let X be a strength random variable following $X_i \sim CD(\alpha_i, \beta), i = 1, 2, 3$ with c.d.f $F_{X_i}(x)$, suppose that X is independent of Y, therefore:

$$G_Y(y) = 1 - e^{\mu\left(1-e^{y^{\beta}}\right)} \qquad y > 0; \mu, \beta > 0 \tag{5}$$

Now, from equation (4):

$$P(X_i > Y) = \int_{y=0}^{\infty} \left(\int_{X_i=y}^{\infty} \alpha_i \beta x^{\beta-1} e^{x^{\beta}} e^{\alpha_i\left(1-e^{x^{\beta}}\right)} dx \right) \mu\beta y^{\beta-1} e^{y^{\beta}} e^{\mu\left(1-e^{y^{\beta}}\right)} dy$$

$$= \int_{y=0}^{\infty}\left(e^{\alpha_i\left(1-e^{y^\beta}\right)}\right)\mu\beta y^{\beta-1}e^{y^\beta}e^{\mu\left(1-e^{y^\beta}\right)}dy$$

$$= \int_{y=0}^{\infty}\mu\beta y^{\beta-1}e^{y^\beta}e^{\alpha_i+\mu\left(1-e^{y^\beta}\right)}dy$$

Similarly, from equation (3), we get:

$$P(X_i > Y) = \frac{\mu}{\alpha_i+\mu} \tag{6}$$

Thus, the system's stress-strength reliability given by:

$$R = \left(\frac{\mu}{\alpha_1+\mu}\right)\left[\frac{\mu}{\alpha_2+\mu}+\frac{\mu}{\alpha_3+\mu}-\left(\frac{\mu}{\alpha_2+\mu}\right)\left(\frac{\mu}{\alpha_3+\mu}\right)\right]$$

$$= \left(\frac{\mu}{\alpha_1+\mu}\right)\left[\frac{(\alpha_3+\mu)\mu+(\alpha_2+\mu)\mu}{(\alpha_2+\mu)(\alpha_3+\mu)}-\frac{\mu^2}{(\alpha_2+\mu)(\alpha_3+\mu)}\right]$$

$$= \left(\frac{\mu}{\alpha_1+\mu}\right)\left[\frac{\alpha_2\mu+\alpha_3\mu+\mu^2}{(\alpha_2+\mu)(\alpha_3+\mu)}\right] = \frac{\alpha_2\mu^2+\alpha_3\mu^2+\mu^3}{(\alpha_1+\mu)(\alpha_2+\mu)(\alpha_3+\mu)} \tag{7}$$

5. Estimation Method

This section uses three different estimating methods to determine the estimator of the stress-strength model's reliability R, and the Chen unknown scale parameters; μ, α_1, α_2 and α_3. These three methods are Maximum Likelihood, Pitman Method, and the Least Squares Method. The best reliability estimate is obtained using these Methods.

5.1. Maximum Likelihood Estimator (MLE)

The most popular method for estimating parameters is the maximum likelihood method [10]. Suppose that $y_1, y_2, \dots, y_n$ are random stress samples with size n following $CD(\mu,\beta)$ in which β is known and μ is an unknown parameter. The MLE function is therefore given by: [11]

$$L(y_1, y_2, \dots, y_n; \mu, \beta) = (\mu\beta)^n \prod_{i=1}^{n} {y_i}^{\beta-1}\, e^{\mu\sum_{i=1}^{n} {y_i}^\beta} e^{\mu\sum_{i=1}^{n}\left(1-e^{{y_i}^\beta}\right)} \tag{8}$$

Then, for equation (8), the natural logarithm function can be expressed as:

$$\ln L = n\ln\mu + n\ln\beta + (\beta-1)\sum_{i=1}^{n}\ln y_i + \sum_{i=1}^{n} {y_i}^\beta + \mu\sum_{i=1}^{n}\left(1-e^{{y_i}^\beta}\right) \tag{9}$$

By differentiating equation (9) w.r.t the unknown parameter μ, and equating its result to zero, we obtain:

$$\frac{\partial \ln L}{\partial\mu} = \frac{n}{\mu} + \sum_{i=1}^{n}\left(1-e^{{y_i}^\beta}\right) \Longrightarrow \frac{n}{\hat{\mu}} + \sum_{i=1}^{n}\left(1-e^{{y_i}^\beta}\right) = 0$$

$$\hat{\mu}_{MLE} = \frac{-n}{\sum_{i=1}^{n}\left(1-e^{{y_i}^\beta}\right)} \tag{10}$$

Where β is known parameter

In the same way, let $x_{11}, x_{21}, \dots, x_{n1}$; $x_{21}, x_{22}, \dots, x_{n2}$ and $x_{31}, x_{32}, \dots, x_{n3}$ are three random strength sample of size n_1, n_2 and n_3 from $CD(\alpha_1,\beta)$; $CD(\alpha_2,\beta)$ and $CD(\alpha_3,\beta)$, respectively. Then, the unknown parameter's MLE estimators of $\alpha_I, I = 1, 2, 3$ are:

$$\hat{\alpha}_{IMLE} = \frac{-n_I}{\sum_{i=1}^{n_I}\left(1-e^{{x_{Ii}}^\beta}\right)} \tag{11}$$

Substitution equations (10), (11) in equation (7) thus produce the MLE estimator for Reliability R, using an invariant property of that method as follows:

$$\hat{R}_{MLE}=\frac{\hat{\alpha}_{2MLE}\hat{\mu}_{MLE}^{2}+\hat{\alpha}_{3MLE}\hat{\mu}_{MLE}^{2}+\hat{\mu}_{MLE}^{3}}{(\hat{\alpha}_{1MLE}+\hat{\mu}_{MLE})(\hat{\alpha}_{2MLE}+\hat{\mu}_{MLE})(\hat{\alpha}_{3MLE}+\hat{\mu}_{MLE})}$$

5.2. Pitman Method (PM)

Let $y_1, y_2, \ldots, y_n$ are stress random sample with size n from $CD(\mu,\beta)$. If the estimate of the scale parameter μ is $\hat{\mu}=h(y_1,y_2,\ldots,y_n)$, then $\hat{\mu}$ must be as follows: [12]

$$\hat{\mu}=\frac{\int_0^{\infty}\frac{1}{\mu^2}L(y_1,y_2,\ldots,y_n;\mu,\beta)d\mu}{\int_0^{\infty}\frac{1}{\mu^3}L(y_1,y_2,\ldots,y_n;\mu,\beta)d\mu} \quad (12)$$

Now based on Pitman method,

By substituting equation (8) into equation (12), the Pitman estimator $\hat{\mu}_{PM}$ of the scale parameter μ will be as follows:

$$\hat{\mu}_{PM}=\frac{\int_0^{\infty}\frac{1}{\mu^2}(\mu\beta)^n\prod_{i=1}^{n}y_i{}^{\beta-1}e^{\mu\sum_{i=1}^{n}y_i{}^{\beta}}e^{\mu\sum_{i=1}^{n}\left(1-e^{y_i{}^{\beta}}\right)}d\mu}{\int_0^{\infty}\frac{1}{\mu^3}(\mu\beta)^n\prod_{i=1}^{n}y_i{}^{\beta-1}e^{\mu\sum_{i=1}^{n}y_i{}^{\beta}}e^{\mu\sum_{i=1}^{n}\left(1-e^{y_i{}^{\beta}}\right)}d\mu}$$

$$\hat{\mu}_{PM}=\frac{\int_0^{\infty}\mu^{n-2}e^{\mu\sum_{i=1}^{n}\left(1-e^{y_i{}^{\beta}}\right)}d\mu}{\int_0^{\infty}\mu^{n-3}e^{\mu\sum_{i=1}^{n}\left(1-e^{y_i{}^{\beta}}\right)}d\mu}$$

Let $-u=\mu\sum_{i=1}^{n}\left(1-e^{y_i{}^{\beta}}\right) \Longrightarrow -du=\sum_{i=1}^{n}\left(1-e^{y_i{}^{\beta}}\right)d\mu$, then we get:

$$\hat{\mu}_{PM}=\frac{\frac{(-1)^{n-2}}{\left(\sum_{i=1}^{n}\left(1-e^{y_i{}^{\beta}}\right)\right)^{n-1}}\int_0^{\infty}u^{n-2}e^{-u}du}{\frac{(-1)^{n-3}}{\left(\sum_{i=1}^{n}\left(1-e^{y_i{}^{\beta}}\right)\right)^{n-2}}\int_0^{\infty}u^{n-3}e^{-u}du}$$

By Gamma function $\int_0^{\infty}x^{\alpha-1}e^{-\beta x}dx=\beta^{\alpha}\Gamma(\alpha)$ [13], we have:

$$\hat{\mu}_{PM}=\frac{\frac{(-1)^{n-2}}{\left[\sum_{i=1}^{n}\left(1-e^{y_i{}^{\beta}}\right)\right]^{n-1}}\frac{\Gamma(n-1)}{(-1)^{n-1}}}{\frac{(-1)^{n-3}}{\left[\sum_{i=1}^{n}\left(1-e^{y_i{}^{\beta}}\right)\right]^{n-2}}\frac{\Gamma(n-2)}{(-1)^{n-2}}}$$

$$=\frac{(-1)^{n-2}\Gamma(n-1)}{(-1)^{n-1}\left[\sum_{i=1}^{n}\left(1-e^{y_i{}^{\beta}}\right)\right]^{n-1}}\;\frac{(-1)^{n-2}\left[\sum_{i=1}^{n}\left(1-e^{y_i{}^{\beta}}\right)\right]^{n-2}}{(-1)^{n-3}\Gamma(n-2)}$$

$$=\frac{-(n-2)}{\sum_{i=1}^{n}\left(1-e^{y_i{}^{\beta}}\right)} \quad (13)$$

Similarly the pitman estimators for the unknown parameters $\alpha_I, I=1,2,3$ are given by:

$$\hat{\alpha}_{IPM}=\frac{-(n_I-2)}{\sum_{i=1}^{n}\left(1-e^{x_{Ii}{}^{\beta}}\right)} \quad (14)$$

Substitution equations (13), (14) in equation (7), the approximate estimator for R can be obtained as bellow:

$$\hat{R}_{PM}=\frac{\hat{\alpha}_{2PM}\hat{\mu}_{PM}^{2}+\hat{\alpha}_{3PM}\hat{\mu}_{PM}^{2}+\hat{\mu}_{PM}^{3}}{(\hat{\alpha}_{1PM}+\hat{\mu}_{PM})(\hat{\alpha}_{2PM}+\hat{\mu}_{PM})(\hat{\alpha}_{3PM}+\hat{\mu}_{PM})}$$

5.3. Least Square Method (LS)

Swain, Venkatraman, and Wilson originally proposed the least squares method in 1988 to estimate the beta distribution's parameters. The process for minimizing the sum of the value and its expected value gives the least square estimators [14]. Suppose that $y_{(1)}, y_{(2)}, \dots, y_{(n)}$ is the order statistics stress random sample with size n from $CD(\mu, \beta)$. The following equation can be minimized to get least square estimator:

$$S = \sum_{i=1}^{n} \left[F\left(y_{(i)}\right) - E\left(F\left(y_{(i)}\right)\right)\right]^2 \tag{15}$$

Where $E\left(F\left(y_{(i)}\right)\right) = Pi$ the plotting position and $P_i = \frac{i}{n+1}$, $i = 1,2, \dots, n$

Putting the cdf of CD in equation (15), we get:

$$S = \sum_{i=1}^{n} \left[1 - e^{\mu\left(1-e^{y_{(i)}^{\beta}}\right)} - Pi\right]^2 \tag{16}$$

So then,

$$S = \sum_{i=1}^{n} \left[\mu\left(1 - e^{y_{(i)}^{\beta}}\right) - q_i\right]^2 \tag{17}$$

Where $q_i = \ln\left(1 - F\left(y_{(i)}\right)\right) = \text{n}(1 - P_i)$

By the partial derivative of equation (17) w.r.t an unknown scale parameter μ and then equating its result to zero, we will obtain:

$$\frac{\partial S}{\partial \mu} = 2\sum_{i=1}^{n} \left[\mu\left(1 - e^{y_{(i)}^{\beta}}\right) - qi\right]\left(1 - e^{y_{(i)}^{\beta}}\right)$$

$$\Rightarrow \hat{\mu}\sum_{i=1}^{n} \left(1 - e^{y_{(i)}^{\beta}}\right)^2 - \sum_{i=1}^{n} q_i\left(1 - e^{y_{(i)}^{\beta}}\right) = 0$$

$$\hat{\mu}_{LS} = \sum_{i=1}^{n} q_i\left(1 - e^{y_{(i)}^{\beta}}\right) \Big/ \sum_{i=1}^{n} \left(1 - e^{y_{(i)}^{\beta}}\right)^2 \tag{18}$$

Similarly, the lest square estimators of unknown parameters $\alpha_I, I = 1, 2, 3$ are given by:

$$\hat{\alpha}_{ILS} = \sum_{i=1}^{n_I} q_i\left(1 - e^{x_{I(i)}^{\beta}}\right) \Big/ \sum_{i=1}^{n_I} \left(1 - e^{x_{I(i)}^{\beta}}\right)^2 \tag{19}$$

Substitution equations (18), (19) in equation (7), we get the LS estimator of the reliability R, approximately will be as:

$$\hat{R}_{LS} = \frac{\hat{\alpha}_{2LS}\hat{\mu}_{LS}^2 + \hat{\alpha}_{3LS}\hat{\mu}_{LS}^2 + \hat{\mu}_{LS}^3}{(\hat{\alpha}_{1LS} + \hat{\mu}_{LS})(\hat{\alpha}_{2LS} + \hat{\mu}_{LS})(\hat{\alpha}_{3LS} + \hat{\mu}_{LS})}$$

6. Simulation study

In this section, a simulation experiment is used to find the best estimate for the reliability of unknown parameters for Chen distribution. All three estimates via the maximum likelihood, Pitman method, and least square method are performed and evaluated according to mean square error criteria (MSE) for three distinct experiments in each instance for the value of the parameter β, using various sample sizes (15, 30, 90) and ($\beta = 4, 0.2, 1.8$).

Using MATLAB 2020, a simulation study is carried out for the nine experiments in order to compare the reliability estimator's performance through the following steps:

Step1: we use the inverse function of equation (1) for generating random values for the random variables by using the following formula: $x = \left[\ln\left(1 - \left(\ln\left(1 - F(x)\right)/\alpha\right)\right)\right]^{1/\beta}$

Step2: The mean square error criteria are used to compare estimate methods: MSE$=\frac{1}{N}\sum_{i=1}^{N}\left(\hat{R}_i - R\right)^2$, where N represents the 1000 replications for each experiment.

The results are listed in the tables from 1 to 3. The MSE values are used to compare the performance of these estimators; for each experiment in the three tables, the MSE value decreases as sample sizes increases for MLE, PM, and LS, In experiments 1 and 3, the MLE estimator has the best MSE value, followed by PM and LS; in experiment 2, the estimated PM has the best MSE value, followed by MLE and LS. As a result, estimator performance for the maximum likelihood is much better than that for the Pitman Method and least square for the most of experiments and all sample sizes.

Table 1: Estimate for Reliability when $\beta = 4$

Exp. 1: $\mu = 1.2, \alpha_1 = 0.6, \alpha_2 = 0.6, \alpha_3 = 0.6$, $R = 0.5926$				
n, n_I		MLE	PM	LS
15,15	MSE	0.0082	0.0112	0.0099
30,30	MSE	0.0045	0.0052	0.0055
90,90	MSE	0.0014	0.0014	0.0017
30,15	MSE	0.0056	0.0065	0.0064
15,90	MSE	0.0064	0.0085	0.0087
30,90	MSE	0.0034	0.0039	0.0044
Exp. 2: $\mu = 0.5, \alpha_1 = 0.8, \alpha_2 = 0.8, \alpha_3 = 0.8$, $R = 0.2390$				
15,15	MSE	0.0066	0.0062	0.0075
30,30	MSE	0.0032	0.0032	0.0039
90,90	MSE	0.0010	0.0010	0.0013
30,15	MSE	0.0035	0.0036	0.0047
15,90	MSE	0.0052	0.0047	0.0054
30,90	MSE	0.0025	0.0024	0.0029
Exp. 3: $\mu = 3, \alpha_1 = 3, \alpha_2 = 3, \alpha_3 = 3$, $R = 0.3750$				
15,15	MSE	0.0091	0.0100	0.0104
30,30	MSE	0.0046	0.0048	0.0055
90,90	MSE	0.0015	0.0015	0.0018
30,15	MSE	0.0053	0.0058	0.0066
15,90	MSE	0.0069	0.0074	0.0081
30,90	MSE	0.0035	0.0038	0.0043

Table 2: Estimate for Reliability when $\beta = 0.2$

Exp. 1: $\mu = 1.2, \alpha_1 = 0.6, \alpha_2 = 0.6, \alpha_3 = 0.6$, $R = 0.5926$				
n, n_I		MLE	PM	LS
15,15	MSE	0.0081	0.0107	0.0096
30,30	MSE	0.0043	0.0049	0.0052
90,90	MSE	0.0015	0.0016	0.0020
30,15	MSE	0.0057	0.0065	0.0068
15,90	MSE	0.0061	0.0081	0.0085
30,90	MSE	0.0033	0.0038	0.0044

Exp. 2: $\mu = 0.5, \alpha_1 = 0.8, \alpha_2 = 0.8, \alpha_3 = 0.8,$ $R = 0.2390$				
15,15	MSE	0.0065	0.0060	0.0075
30,30	MSE	0.0032	0.0031	0.0039
90,90	MSE	0.0010	0.0010	0.0013
30,15	MSE	0.0039	0.0039	0.0053
15,90	MSE	0.0048	0.0045	0.0052
30,90	MSE	0.0025	0.0025	0.0030
Exp. 3: $\mu = 3, \alpha_1 = 3, \alpha_2 = 3, \alpha_3 = 3,$ $R = 0.3750$				
15,15	MSE	0.0089	0.0096	0.0104
30,30	MSE	0.0045	0.0046	0.0054
90,90	MSE	0.0015	0.0015	0.0018
30,15	MSE	0.0058	0.0061	0.0073
15,90	MSE	0.0065	0.0070	0.0078
30,90	MSE	0.0036	0.0038	0.0044

Table 3: Estimate for Reliability when $\beta = 1.8$

Exp. 1: $\mu = 1.2, \alpha_1 = 0.6, \alpha_2 = 0.6, \alpha_3 = 0.6,$ $R = 0.5926$				
n, n_I		MLE	PM	LS
15,15	MSE	0.0090	0.0122	0.0103
30,30	MSE	0.0043	0.0049	0.0053
90,90	MSE	0.0014	0.0015	0.0019
30,15	MSE	0.0061	0.0071	0.0071
15,90	MSE	0.0060	0.0082	0.0082
30,90	MSE	0.0033	0.0038	0.0045
Exp. 2: $\mu = 0.5, \alpha_1 = 0.8, \alpha_2 = 0.8, \alpha_3 = 0.8,$ $R = 0.2390$				
15,15	MSE	0.0061	0.0061	0.0075
30,30	MSE	0.0033	0.0032	0.0040
90,90	MSE	0.0011	0.0011	0.0014
30,15	MSE	0.0038	0.0038	0.0052
15,90	MSE	0.0053	0.0048	0.0055
30,90	MSE	0.0026	0.0025	0.0030
Exp. 3: $\mu = 3, \alpha_1 = 3, \alpha_2 = 3, \alpha_3 = 3,$ $R = 0.3750$				
15,15	MSE	0.0086	0.0098	0.0104
30,30	MSE	0.0046	0.0048	0.0056
90,90	MSE	0.0016	0.0016	0.0020
30,15	MSE	0.0056	0.0060	0.0072
15,90	MSE	0.0071	0.0074	0.0082
30,90	MSE	0.0036	0.0038	0.0045

7. Conclusion

In this study, three methods are presented to estimate the multicomponent stress-strength of coherent system reliability that has different parameters, based on Chen Distribution. The estimator performance for the maximum likelihood is much better than that of the Pitman Method, and least square for the most of experiments and all sample sizes, based on simulation results that have appeared.

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