Mustansiriyah Journal of Pure and Applied Sciences, Print ISSN: 2957-9910. Online ISSN: 2957-9929, Vol. 4, No. 2, 2026, Pages 103-110
DOI: 10.47831/mjpas.v4i2.372

MJPAS

MUSTANSIRIYAH JOURNAL OF PURE AND APPLIED SCIENCES

Journal homepage:
https://mjpas.uomustansiriyah.edu.iq/index.php/mjpas

RESEARCH ARTICLE - MATHEMATICS

New Generalized Structures of Supra Compact and Supra Lindelöf Spaces

Maymona Rasheed Khalaf [1], Alaa. M. F. AL. Jumaili [2*]

[1,2] Department of mathematics, College of Education for pure science, University of Anbar-Iraq

[1] may23u2008@uoanbar.edu.iq [2*] eps.alaamahmood.farhan@uoanbar.edu.iq

Article Info.

Article history:

Received
24 December 2024

Accepted
6 March 2025

Publishing
30 June 2026

Abstract

The main aim of the present paper is to apply a new class of generalized supra open set namely, supra δ-ß-open to investigate and study other novel extended structures of supra compact and Lindelöf spaces namely, supra δ-ß-compact (supra δ-ß-Lindelöf) spaces, almost supra δ-ß-compact (almost supra δ-ß-Lindelöf) spaces and mildly supra δ-ß-compact (mildly supra δ-ß-Lindelöf) spaces in supra topological spaces. Various essential properties and fundamental characterizations for each class are verified; as well some interesting and appropriate examples are presented to demonstrate the relationships among these kinds of spaces. Additionally, a novel notion of supra limit points called, supra δ-ß-limit points is defined and some basic properties related to this concept are investigated and proved, also various arousing interest results are developed which may be importance of other future studies.

The official journal published by the College of Education at Mustansiriyah University

Keywords: Supra δ-ß-open set, supra δ-ß-compact, supra δ-ß-Lindelöf, almost-(mildly) supra δ-ß-compact, almost-(mildly) supra δ-ß-Lindelöf.

1. Introduction

The classes of extended supra compact and Lindelöf spaces are very beneficial and crucial classes of supra topological spaces, as well in other branches of pure and applied mathematics. They are playing a vital role in the study of closed and bounded intervals of real numbers, spaces of continuous mappings, optimizations, nets, filters and differential equations . Many researchers have been suggest various extensions of supra open sets, including: supra α-open, supra pre-open, supra semi-open, supra b-open, supra ß-open sets and so on, which are playing a vital role to obtain many innovative outcomes analogy to general topological spaces. In 1974 R. Staum [1] investigated two weaker forms of compactness and Lindelöfness namely, mildly compact and mildly Lindelöf spaces. After one year, other kinds of compactness namely, almost compact spaces were presented via P. H. Lambrinos [2]. After the appearance the classes of extended open sets, many kinds of compactness, Lindelöfness, mildly compactness and almost compactness have been devised and investigated, [3] with the references cited within. Subsequently, various characteristics of strongly compactness, Lindelöfness and $\mathcal{P}$-closed spaces were discussed via the researchers [4, 5]. Next, some descriptions of extensions almost soft compactness, almost soft Lindelöfness and soft closed spaces have been presented and studied via the authors [6, 7]. Mashhour *et al.* [8] developed and studied some topological properties from the standpoint of supra topology via neglecting an intersection condition of topology and this case makes supra topology more flexible to describe various real life problems. Motivated via this fact, many authors have studied various classical topological classes including interior, closure and limit points, as well compactness, Lindelöfness, separation axioms and extension open sets on the supra topological spaces. For more information we refer the authors to [9-15]. On the other hand, supra topological spaces was collective with a partial order relation to construct a novel mathematical structure, called supra topological ordered spaces, and based on this fact many topological structures have been presented and studied, we refer the authors to [16-20] . The main goal of our manuscript is to present and study three new classes of extended supra compact and Lindelöf spaces in supra topological spaces. These classes are defined by employing a new class of generalized supra open sets namely, supra δ-ß-open. As well, the relationships among these kinds of spaces are illustrated by some suitable examples. In addition, numerous of fundamental characterizations related to these kinds of supra compactness and Lindelöfness are established. Also, the class of supra δ-ß-limit points is defined and some basic properties related to this notion are discussed and proved.

2. Materials and Methods

During this work, $(\mathcal{X},\mathcal{T}),(\mathcal{Y},\mathcal{T}^*)$ and $(\mathcal{Z},\mathcal{T}^{**})$ (Concisely, $\mathcal{X},\mathcal{Y}$ & Z) symbolized to topological spaces. Recollect the next required definitions and results of extended supra open sets which help us to study our main outcomes in the sequels.

Definition 2.1: [11] A sub family μ of a power set $\mathcal{P}(\mathcal{X})$ of $\mathcal{X}$ is said to be supra topology (concisely, Sup-Top) if it's closed under arbitrary union and $\mathcal{X}$is a member of μ. In that case, the pair $(\mathcal{X},\mu)$ is called Sup-Top-Sp.

Remark 2.2: [11] The members of μ are called Sup-open sets and the complement of Sup-open set is called Sup-closed set, as well the complement of $\mathcal{H} \subseteq \mathcal{X}$ is denoted via $\mathcal{X} \,/\, \mathcal{H}$.

Definition 2.3: [11] Let $(\mathcal{X},\mu)$ be Sup-Top-Sp and $\mathcal{H} \subseteq \mathcal{X}$, so:

The closure and interior of $\mathcal{H} \subseteq \mathcal{X}$ will be indicated via $Cl^{\mu}(\mathcal{H})$ & $Int^{\mu}(\mathcal{H})$ *resp*, and are described as:

$(\boldsymbol{a})Cl^{\mu}(\mathcal{H}) = \bigcap\{\mathcal{M} \subseteq \mathcal{X}: \mathcal{M} \text{ is Sup} - \text{closed and } \mathcal{H} \subseteq \mathcal{M}\}$.

$(\boldsymbol{b})Int^{\mu}(\mathcal{H}) = \bigcup\{\mathcal{M} \subseteq \mathcal{X}: \mathcal{M} \text{ is Sup} - \text{ open and } \mathcal{M} \subseteq \mathcal{H}\}$.

Definition 2.4: [11] Let $\mathcal{X}$ be Top-Sp and μ Sup-Top, so μ is Sup-Top associated with $\mathcal{T}$if $\mathcal{T} \subseteq \mu$.

Definition 2.5: [21] A subset $\mathcal{H} \subseteq \mathcal{X}$ is called δ-ß-open if $\mathcal{H} \subseteq Cl(Int(Cl_{\delta}(\mathcal{H})))$.

Definition 2.6: [11] If $(\mathcal{X},\mathcal{T})$ & $(\mathcal{Y},\mathcal{T}^*)$ are two Top-Sps and μ is an associated Sup-Top with $\mathcal{T}$. Then, $\mathfrak{F}:(\mathcal{X},\mathcal{T}) \longrightarrow (\mathcal{Y},\mathcal{T}^*)$ is called Sup-continuous (briefly, Cont) if the inverse image of every open subset in$(\mathcal{Y},\mathcal{T}^*)$ is supra open subset in$(\mathcal{X},\mu)$.

Definition 2.7: [9] A Sup-Top-Sp-$(\mathcal{X},\mu)$ is said to be:

$(\boldsymbol{i})$Sup-compact(*resp*. Sup-Lindelöf) supplied that each Sup-open-cover of $\mathcal{X}$ has finite (*resp*. countable) sub cover.

$(\boldsymbol{ii})$Almost Sup-compact(resp. Almost Sup-Lindelöf) supplied that each Sup-open-cover of $\mathcal{X}$ has finite (resp. countable) sub-cover which the Sup-closures of whose members cover$\mathcal{X}$.

$(\boldsymbol{iii})$Mildly Sup-compact(resp. mildly Sup-Lindelöf) provided that each Sup-clopen-cover of $\mathcal{X}$ has finite (*resp*. countable) sub-cover.

Definition2.8: [22] Assume that $\mathfrak{F}:(\mathcal{X},\mathcal{T}) \longrightarrow (\mathcal{Y},\mathcal{T}^*)$, with $\{\mathcal{H}_i: i \in \mho\}$ & $\{\mathcal{M}_i: i \in \mho\}$ be collections of subsets of $\mathcal{X}$ & $\mathcal{Y}$, *resp*. So,

$(\boldsymbol{i})$ $\mathfrak{F}(\bigcup_{i\in\mho}\mathcal{H}_i) = \bigcup_{i\in\mho}\mathfrak{F}(\mathcal{H}_i)$.

$(\boldsymbol{ii})$ $\mathfrak{F}^{-\mathbf{1}}(\bigcup_{i\in\mho}\mathcal{M}_i) = \bigcup_{i\in\mho}\mathfrak{F}^{-\mathbf{1}}(\mathcal{M}_i)$.

Definition 2.9: [22] For $\mathcal{X} \neq \emptyset$, a sub-collection $\{\mathcal{D}_i: i \in \mho\}$ of $2^{\mathcal{X}}$ is have:

$(\boldsymbol{i})$ Finite intersection property (Concisely, FIP) if each finite subcollection $\{\mathcal{D}_i: i \in \mho\}$ has nonempty intersection.

$(\boldsymbol{ii})$ Countable intersection property(Concisely, CIP) if each countable subcollection $\{\mathcal{D}_i: i \in \mho\}$ has nonempty intersection.

3- Supra-*δ*-*ß*-Compact and Supra-*δ*-*ß*-Lindelöf spaces

This segment devoted to introduce new kinds of extended supra compact and Sup-Lindelöf-sp based on novel extended of Sup-open sets and study the relationships among them, as well the equivalent conditions for each one of them are presented.

Definitions 3.1: $\mathcal{H} \subseteq \mathcal{X}$ is said to be:

$(\boldsymbol{i})$ Sup-*δ*-*ß*-open set if $\mathcal{H} \subseteq Cl^{\mu}\left(Int^{\mu}\left(Cl^{\mu}_{\delta}(\mathcal{H})\right)\right)$. The complement of Sup-*δ*-*ß*-open is Sup-*δ*-*ß*-closed.

$(\boldsymbol{ii})$ Sup-*δ*-*ß*-closed if $Int^{\mu}\left(Cl^{\mu}\left(Int^{\mu}_{\delta}(\mathcal{H})\right)\right) \subseteq \mathcal{H}$.

Definition 3.2: If $(\mathcal{X},\mathcal{T})$ and $(\mathcal{Y},\mathcal{T}^*)$ are two Top-Sp and μ is an associated Sup-Top with $\mathcal{T}$. So, $\mathfrak{F}:(\mathcal{X},\mathcal{T}) \longrightarrow (\mathcal{Y},\mathcal{T}^*)$ is called Sup-*δ*-*ß*-Cont if the inverse image of each open subset of $(\mathcal{Y},\mathcal{T}^*)$ is Sup-*δ*-*ß*-open of $(\mathcal{X},\mu)$.

Example 3.3: Suppose $\mathcal{X} = \{u,v,w\}$ and $\mathcal{T} = \{\mathcal{X},\emptyset,\{u,v\}\}$ is topology on $\mathcal{X}$. The Sup-Top μ is described as: $\mu = \{\mathcal{X},\emptyset,\{u\},\{v\},\{u,v\}\}$. Presume that $\mathfrak{F}:(\mathcal{X},\mathcal{T}) \longrightarrow (\mathcal{X},\mathcal{T})$ described as: $\mathfrak{F}(u) = v, \mathfrak{F}(v) = w, \mathfrak{F}(w) = u$. In that case, $\mathfrak{F}$ is Sup-*δ*-*ß*- Cont map, because the inverse image of the open set $\{u,v\}$ is $\{u,w\}$ which is Sup- *δ*-*ß*-open of $(\mathcal{X},\mu)$.

Definition 3.4: Assume $(\mathcal{X},\mu)$ is Sup-Top-Sp and $\mathcal{H} \subseteq \mathcal{X}$ of $\mathcal{X}$ so:

The Sup-*δ*-*ß*-closure and Sup-*δ*-*ß*-interior $\mathcal{H}$ symbolized via $Cl^{\mu}_{\delta-\beta}(\mathcal{H})$ & $Int^{\mu}_{\delta-\beta}(\mathcal{H})$, *resp*, and are described as follows:

$(\boldsymbol{a})\ Cl^{\mu}_{\delta-\beta}(\mathcal{H}) = \bigcap\{\mathcal{M} \subseteq \mathcal{X}: \mathcal{M} \text{ is Sup } \delta-\beta-\text{closed and } \mathcal{H} \subseteq \mathcal{M}\}$.

$(\boldsymbol{b}) Int^{\mu}_{\delta-\beta}(\mathcal{H}) = \bigcup\{\mathcal{M} \subseteq \mathcal{X}: \mathcal{M} \text{ is Sup}\delta-\beta-\text{open and } \mathcal{M} \subseteq \mathcal{H}\}$.

Definitions 3.5: A collection $\{\mathcal{D}_i: i \in \mho\}$ of Sup-*δ*-*ß*-open sets in $(\mathcal{X},\mu)$ is called Sup-*δ*-*ß*-open-cover of $\mathcal{W} \subseteq \mathcal{X}$ if $\mathcal{W} \subseteq \bigcup\{\mathcal{D}_i: i \in \mho\}$.

Definitions 3.6: A $(\mathcal{X},\mu)$ is said to be:

$(\boldsymbol{a})$ Supra *δ*-*ß*-compact (briefly, Sup-*δ*-*ß*-comp) if each Sup-*δ*-*ß*-open cover of $\mathcal{X}$ has finite sub-cover.

$(\boldsymbol{b})$ Supra δ-*ß*-Lindelöf (briefly, Sup-*δ*-*ß*-Linde) if each Sup-*δ*-*ß*-open cover of $\mathcal{X}$ has countable sub-cover.

Definitions 3.7: A subset $\mathcal{W}$ of Sup-Top-Sp-$(\mathcal{X},\mu)$ is called Sup-*δ*-*ß*-comp-(*resp*. Sup-*δ*-*ß*-Linde) relative to $\mathcal{X}$ if each Sup-*δ*-*ß*-op-cover of $\mathcal{W}$, reducible to finite (*resp*.countable) sub-cover.

Next, present some examples; first one satisfies class of Sup-δ-ß-compactness and second one doesn't:

Example3.8: Assume $\mu = \{\emptyset, \mathcal{D}_m \subseteq \mathbb{N}: \mathcal{D}_m = \{m, m+1, m+2, \dots\dots: m \in \mathbb{N}\}\}$ is Sup-Top on $\mathbb{N}$. As, infimum $(\mathcal{D}_m) = m, \forall$ subset $\mathcal{D}_m$of $(\mathbb{N},\mu)$, in that case $(\mathbb{N},\mu)$ is Sup-*δ*-*ß*-comp-sp.

Example 3.9: Presume $\mu = \{\emptyset, \mathcal{D} \subseteq \mathbb{N}: \exists\, p \in \mathcal{D}\ (s.t)\ 2 \,\big|\, p\}$ is Sup-Top on $\mathbb{N}$. At this instant, $\nabla = \{\{m, m+1\}: m \in \mathbb{N}\}$ is Sup-*δ*-*ß*-open-cover of $\mathbb{N}$. As ∇ has not finite sub-cover, in that case $(\mathbb{N},\mu)$ is not Sup-*δ*-*ß*-comp-sp.

Proposition 3.10: Every Sup-*δ*-*ß*-comp-space is Sup-*δ*-*ß*-Linde-space.

Proof: It is simple and straightforward, thus omitted.

Remark 3.11: Reverse of Proposition 3.10, is not correct in general as appearance in Example3.9.

Proposition3.12: Every finite (*resp*. countable) Sup-Top-Sp-$(\mathcal{X},\mu)$ is Sup-*δ*-*ß*-comp-(*resp*. Sup-*δ*-*ß*-Linde).

Proof: Firstly, if $\mathcal{X}$ finite, so the evidence follows promptly from the truth $\mathcal{P}(\mathcal{X})$ finite.

Secondly, If $\mathcal{X}$ countable, in that case the largest cover with regard to the number of its element is $\nabla = \big\{\{p\}: p \in \mathcal{X}\big\}$. Countability for ∇ completes evidence.

Proposition3.13: Finite (*resp*. countable) union of Sup-*δ*-*ß*-comp-subsets of Sup-Top-Sp $(\mathcal{X},\mu)$ is Sup-*δ*-*ß*-comp-(*resp*. Sup-*δ*-*ß*-Lind).

Proof: It is simple and straightforward, thus deleted.

Proposition 3.14: Each Sup-*δ*-*ß*-comp-(*resp*. supra *δ*-*ß*-Linde)-sp $(\mathcal{X},\mu)$ is Sup-comp-(*resp*. Sup-*δ*-*ß*-Linde).

Proof: It is simply got from truth that each Sup-open is Sup-*δ*-*ß*-open.

Remark 3.15: The converse of the above Proposition is not always correct in general as appearance in the next example:

Example 3.16: Assume $\mu = \{\emptyset, \mathbb{R}, \{1,2\}, \{2,3\}, \{1,2,3\}\}$ is Sup-Top on $\mathbb{R}$. Apparently,$(\mathbb{R},\mu)$ is Sup-comp. On the other hand a family $\nabla = \{\{1, 3, p\}: p \in \mathbb{R}\}$ is Sup-*δ*-*ß*-open-cover for $(\mathbb{R},\mu)$. As, ∇ hasn't countable-subcover of $\mathbb{R}$, in that case $(\mathbb{R},\mu)$ isn't Sup-*δ*-*ß*-Linde-sp.

Theorem 3.17: Each Sup-*δ*-*ß*-closed subset of Sup-*δ*-*ß*-comp-(*resp*. Sup-*δ*-*ß*-Linde)-Sp $(\mathcal{X},\mu)$ is Sup-*δ*-*ß*-comp-(*resp*. Sup-*δ*-*ß*-Linde).

Proof: First only prove the theorem in case of Sup-*δ*-*ß*-comp-sp, the other case for Sup-*δ*-*ß*-Linde-sp can be made similarly.

Presume $\mathcal{K}$ is Sup-*δ*-*ß*-closed subset of $\mathcal{X}$ with $\{\mathcal{D}_i: i \in \mho\}$ is Sup-*δ*-*ß*-open-cover of $\mathcal{K}$. In that case $\mathcal{K}^c$ is Sup-*δ*-*ß*-open and $\mathcal{K} \subseteq \bigcup_{i\in\mho}\mathcal{D}_i$. Consequently, $\mathcal{X} = \bigcup_{i\in\mho}\mathcal{D}_i \cup \mathcal{K}^c$. As, $\mathcal{X}$ is Sup-*δ*-*ß*-comp, so $\mathcal{X} = \bigcup_{i=1}^{i=m}\mathcal{D}_i \cup \mathcal{K}^c$. Therefore, $\mathcal{K} \subseteq \bigcup_{i=1}^{i=m}\mathcal{D}_i$. consequently, $\mathcal{K}$ is Sup-*δ*-*ß*-comp-set.

Theorem 3.18: A Sup-Top-Sp-$(\mathcal{X},\mu)$ is Sup-*δ*-*ß*-comp-(*resp*. Sup-*δ*-*ß*-Linde) *iff* each collection of Sup-*δ*-*ß*-closed subsets of $\mathcal{X}$, satisfying FIP (*resp*. CIP), has itself nonempty intersection.

Proof: First only verify the theorem in case of Sup-*δ*-*ß*-comp-sp, the other case for Sup-*δ*-*ß*-Linde-sp can be made similarly.

Necessity: Suppose $\nabla = \{\mathcal{K}_i: i \in \mho\}$ is family of Sup-*δ*-*ß*-closed subsets of $\mathcal{X}$ which has the FIP. Presume $\bigcap_{i\in\mho}\mathcal{K}_i = \emptyset$. In that case $\mathcal{X} = \bigcup_{i\in\mho}\mathcal{K}_i^c$. as $\mathcal{X}$ is Sup-*δ*-*ß*-comp, so $\mathcal{X} = \bigcup_{i=1}^{i=m}\mathcal{K}_i^c$. Consequently, $\bigcap_{i=1}^{i=m}\mathcal{K}_i = \emptyset$. But this contradicts ∇ has FIP. so, ∇ has non-empty intersection.

Sufficiency: Assume $\Omega = \{\mathcal{D}_i: i \in \mho\}$ is Sup-*δ*-*ß*-op-cover of $\mathcal{X}$. Presume, Ω has no finite sub-cover. So, $\mathcal{X} \setminus \bigcup_{i=1}^{i=m}\mathcal{D}_i \neq \emptyset, \forall m \in \mathbb{N}$. Now, $\bigcap_{i=1}^{i=m}\mathcal{D}_i^c \neq \emptyset$ implies $\{\mathcal{D}_i^c: i \in \mho\}$ is collection of Sup-*δ*-*ß*-closed subset which has FIP. Therefore $\bigcap_{i\in\mho}\mathcal{D}_i^c \neq \emptyset$. Thus, $\mathcal{X} \neq \bigcup_{i\in\mho}\mathcal{D}_i$. But this contradictions Ω is Sup-*δ*-*ß*-op-cover of $\mathcal{X}$. For this reason $(\mathcal{X},\mu)$ Sup-*δ*-*ß*-comp.

Theorem 3.19: Let $\mathcal{H}$ be Sup-*δ*-*ß*-comp-(*resp*. supra *δ*-*ß*-Linde) subset of $\mathcal{X}$ and $\mathcal{M}$ be Sup-*δ*-*ß*-closed subset of $\mathcal{X}$, so $\mathcal{H} \cap \mathcal{M}$ be Sup-*δ*-*ß*-comp-(*resp*. Sup-*δ*-*ß*-Linde).

Proof: Suppose $\nabla = \{\mathcal{D}_i : i \in \mho\}$ is Sup-*δ*-*ß*-open-cover of $\mathcal{H} \cap \mathcal{M}$. In that case $\mathcal{H} \subseteq \bigcup_{i\in\mho}\mathcal{D}_i \cup \mathcal{M}^c$. As, $\mathcal{H}$ is Sup-*δ*-*ß*-comp, so there exist countable set $\mathcal{S}$ (s. t) $\mathcal{H} \subseteq \bigcup_{i\in\mathcal{S}}\mathcal{D}_i \cup \mathcal{M}^c$. Consequently, $\mathcal{H} \cap \mathcal{M} \subseteq \bigcup_{i\in\mathcal{S}}\mathcal{D}_i$. Therefore, $\mathcal{H} \cap \mathcal{M}$ is Sup-*δ*-*ß*-Linde.

The other case for Sup-δ-ß-Linde-sp can be made similarly.

Theorem 3.20: The Sup-*δ*-*ß*-Cont image of Sup-*δ*-*ß*-comp-(*resp*. Sup-*δ*-*ß*-Linde) set is comp- (*resp*. *δ*-*ß*-Linde).

Proof: Suppose $\mathcal{G}: \mathcal{X} \longrightarrow \mathcal{Y}$ is Sup-*δ*-*ß*-Cont map and $\mathcal{H}$ is Sup-*δ*-*ß*-comp-subset of $\mathcal{X}$. Assume $\{\mathcal{D}_i : i \in \mho\}$ op-cover of $\mathcal{G}(\mathcal{H})$. Currently, $(\mathcal{H}) \subseteq \bigcup_{i\in\mho}\mathcal{D}_i \Longrightarrow \mathcal{H} \subseteq \bigcup_{i\in\mho}\mathcal{G}^{-1}(\mathcal{D}_i)$. As, $\mathcal{G}$ is Sup-*δ*-*ß*-Cont, in that case $\mathcal{G}^{-1}(\mathcal{D}_i)$ is Sup-*δ*-*ß*-open, $\forall\, i \in \mho$. Via supposition, $\mathcal{H}$is Sup-*δ*-*ß*-comp, so $\mathcal{G}(\mathcal{H}) \subseteq \bigcup_{i=1}^{i=m}\mathcal{D}_i$. Therefore, $\mathcal{G}(\mathcal{H})$ is comp.

By the same method, one can prove theorem in case of Sup-δ-ß-Linde-set.

Definitions 3.21: Let $\mathcal{H} \subseteq (\mathcal{X},\mu)$ and $p \in \mathcal{X}$. A point p is called Sup-*δ*-*ß*-limit point of $\mathcal{H}$ if each Sup-*δ*-*ß*-open set including p includes as a minimum one point of $\mathcal{H}$ diverse than p.

The set of each Sup-*δ*-*ß*-limit point of $\mathcal{H}$ is symbolized via $\mathcal{H}^{\delta\beta\mathcal{L}^{\mu}}$.

Theorem3.22: If $\mathcal{H} \subseteq \mathcal{X}$. Then, $\mathcal{H}$ is Sup-*δ*-*ß*-closed *iff* $\mathcal{H}^{\delta\beta\mathcal{L}^{\mu}} \subseteq \mathcal{H}$.

Proof: **Necessity:** Presume $\mathcal{H}$is Sup-*δ*-*ß*-closed and $p \notin \mathcal{H}$. In that case, $p \in \mathcal{H}^c$. As, $\mathcal{H}^c$ is Sup-*δ*-*ß*-open and $\mathcal{H}^c \cap \mathcal{H} = \emptyset$, so $p \notin \mathcal{H}^{\delta\beta\mathcal{L}^{\mu}}$.Consequently, $\mathcal{H}^{\delta\beta\mathcal{L}^{\mu}} \subseteq \mathcal{H}$.

Sufficiency: Assume $p \in \mathcal{H}^c$ & $\mathcal{H}^{\delta\beta\mathcal{L}^{\mu}} \subseteq \mathcal{H}$, subsequently $p \notin \mathcal{H}^{\delta\beta\mathcal{L}^{\mu}}$. As a result there exists Sup-*δ*-*ß*-open $\mathcal{D}_p$ (s. t) $\mathcal{D}_p \setminus \{p\} \cap \mathcal{H} = \emptyset$. Because, $p \in \mathcal{H}^c$, so $\mathcal{D}_p \cap \mathcal{H} = \emptyset$. Now, $\mathcal{D}_p \subseteq \mathcal{H}^c$. Consequently, $\mathcal{H}^c = \bigcup\{\mathcal{D}_p : p \in \mathcal{H}^c\}$. Hence, $\mathcal{H}^c$ is Sup-*δ*-*ß*-open. For this reason $\mathcal{H}$ is Sup-*δ*-*ß*-closed set.

Corollary 3.23: If $\mathcal{H}$ is subset of Sup-*δ*-*ß*-comp-(*resp*. Sup-*δ*-*ß*-Linde) spaces. So, $\mathcal{H}^{\delta\beta\mathcal{L}^{\mu}}$ is Sup-*δ*-*ß*-comp- (*resp*. Sup-*δ*-*ß*-Linde).

Theorem 3.24: Each infinity subset of Sup-*δ*-*ß*-comp-sp has Sup-*δ*-*ß*-limit point.

Proof: Presume $\boldsymbol{\mathcal{H}}$is infinity subset of Sup-*δ*-*ß*-comp-sp-$(\boldsymbol{\mathcal{X}},\boldsymbol{\mu})$. Assume $\boldsymbol{\mathcal{H}}$ has not Sup-*δ*-*ß*-limit point. In that case $\forall\, \boldsymbol{p_i} \in \boldsymbol{\mathcal{X}}$, there exist Sup-*δ*-*ß*-open $\boldsymbol{\mathcal{D}_{p_i}}$ (s. t) $\boldsymbol{\mathcal{D}_{p_i}} \cap \boldsymbol{\mathcal{H}} \setminus \{\boldsymbol{p_i}\} = \emptyset$. Now, the family $\boldsymbol{\nabla} = \{\boldsymbol{\mathcal{D}_{p_i}} : \boldsymbol{p_i} \in \boldsymbol{\mathcal{X}}\}$ forms Sup-*δ*-*ß*-open-cover of $\boldsymbol{\mathcal{X}}$. As $\boldsymbol{\mathcal{X}}$ is Sup-*δ*-*ß*-comp, so $\boldsymbol{\mathcal{X}} = \bigcup_{\boldsymbol{i=1}}^{\boldsymbol{i=m}}\boldsymbol{\mathcal{D}_{p_i}}$. Consequently, $\boldsymbol{\mathcal{X}}$ has mainly $\boldsymbol{m}$ points of $\boldsymbol{\mathcal{H}}$. Implies $\boldsymbol{\mathcal{H}}$ is finite. Except this contradiction $\boldsymbol{\mathcal{H}}$ is infinite. Hence, $\boldsymbol{\mathcal{H}}$ has Sup-*δ*-*ß*-limit point.

4- Almost Supra-*δ*-*ß*-Compact and Almost Supra-*δ*-*ß*-Lindelöf spaces

In this segment, another extension for the classical meaning of almost Supra-compact (briefly, AL-Sup-Comp) and almost Supra-LindeLöf (briefly, AL-Sup-Linde)-sp has been studied.

Definitions 4.1: A Sup-Top-Space $(\mathcal{X},\mu)$ is said to be AL-Sup-*δ*-*ß*-comp-(*resp*. AL-Sup-*δ*-*ß*-Linde) if each Sup-*δ*-*ß*-open-cover of $\mathcal{X}$ has finite (*resp*. countable) sub-cover consisting of Sup-*δ*-*ß*-closures whose members cover $\mathcal{X}$.

Definitions 4.2: A subset $\mathcal{W} \subseteq \mathcal{X}$ is said to be AL-Sup-*δ*-*ß*-comp-(*resp*. AL-Sup-*δ*-*ß*-Linde) relative to $\mathcal{X}$ if each supra *δ*-*ß*-open-cover of $\mathcal{W}$ has finite (*resp*. countable) sub-cover consisting of Sup-*δ*-*ß*-closures whose members cover $\mathcal{W}$.

Next, we present two examples; the first satisfies the class of AL-Sup-δ-ß-Compactness and the second doesn't:

Example4.3: Presume $\mu = \{\emptyset, \mathcal{D}_m = \{1,2,\dots,2m+1\}\ or\ \mathcal{D}_m = \{1,2,\dots,3m+1\} : m \in \mathbb{N}\}$ is Sup-Top for $\mathbb{N}$. Any nonempty Sup-op-subset in $(\mathbb{N},\mu)$ is dense. It's renowned each nonempty Sup-*δ*-*ß*-open including nonempty Sup-op-set, in that case each non-empty Sup-*δ*-*ß*-open in $(\mathbb{N},\mu)$ is dense. Consequently, $(\mathbb{N},\mu)$ AL-Sup-*δ*-*ß*-comp. On the other hand, if $\nabla = \{\{1,2,3,p\} : p \in \mathbb{N}\ and\ p > 3\}$ is Sup-*δ*-*ß*-op-cover for $\mathbb{N}$. Because ∇ hasn't finite sub-cover, so $(\mathbb{N},\mu)$ isn't Sup-*δ*-*ß*-comp.

Example 4.4: Assume $\mu = \{\emptyset, \mathcal{D} \subseteq \mathbb{Q}\ (\text{s.t})\ \mathcal{D}\ \text{including at least two elements}\}$ is Sup-Top on $\mathbb{Q}$ and assume ∇ is family for each set which has only two members. In that case $(\mathbb{Q},\mu)$ isn't AL-Sup-*δ*-*ß*-comp.

Proposition 4.5: Each AL-Sup-*δ*-*ß*-comp-space is AL-Sup-*δ*-*ß*-Linde-space.

Proof: It is proof is straightforward.

Remark 4.6: The converse of the above Proposition is not true in general as appearance in Example-4.4.

Proposition 4.7: Each Sup-δ-ß-comp-(*resp*. Sup-δ-ß-Linde)-space $(\mathcal{X},\mu)$ is AL-Sup-δ-ß-comp-(*resp*. AL-Sup-δ-ß-Linde).

Proof: It follows directly from the truth that each set is included in it's Sup-δ-ß-closure.

Remark 4.8: Next example demonstrates that the converse of the above Proposition is not always hold.

Example 4.9: Presume $\mu = \{\emptyset, \mathbb{R}, \{1\}, \{2\}, \{1,2\}\}$ is Sup-Top on $\mathbb{R}$. As, each Sup-δ-ß-open set, except of $\{1\}\&\{2\}$, must include Sup-open set $\{1,2\}$, let $\nabla = \{\{1, 2, \wp\}: \wp \in \mathbb{R}\}$ is Sup-δ-ß-open-cover for $(\mathbb{R},\mu)$, clearly ∇ has no countable sub-cover of $\mathbb{R}$. Therefore, $(\mathbb{R},\mu)$ is not Sup-δ-ß-Linde-sp. On the other hand, Cl^{μ}l$(\{1,2\}) = \delta\beta Cl^{\mu}(\{1,2\}) = \mathbb{R}$. Thus, $(\mathbb{R},\mu)$ is AL-Sup-δ-ß-comp-sp.

We recall the following Propositions with direct proofs and so omitted.

Proposition 4.10: Each finite (*resp*. countable) Sup-Top-Sp $(\mathcal{X},\mu)$ is AL-Sup-δ-ß-comp-(*resp*. AL-Sup-δ-ß-Lindel).

Proposition4.11: Finite (*resp*. countable) union of AL-Sup-δ-ß-comp-subsets of Sup-Top-Space $(\mathcal{X},\mu)$ is AL-Sup-δ-ß-compact (*resp*. Sup-δ-ß-Lindelöf).

Definitions 4.12: $\mathcal{W} \subseteq \mathcal{X}$ is called Sup-δ-ß-clopen if $\mathcal{W}$ is Sup-δ-ß-open and Sup-δ-ß-closed.

Remark 4.13: A subset $\mathcal{W} \subseteq (\mathcal{X},\mu)$ is said to be Sup-δ-ß-clopen *iff* $Int^{\mu}\left(Cl^{\mu}\left(Int^{\mu}_{\delta}(\mathcal{H})\right)\right) \subseteq \mathcal{H} \subseteq Cl^{\mu}\left(Int^{\mu}\left(Cl^{\mu}_{\delta}(\mathcal{H})\right)\right)$.

Theorem 4.14: Every Sup-δ-ß-clopen subset of AL-Sup-δ-ß-comp-(*resp*. AL-Sup-δ-ß-Linde)-Space $(\mathcal{X},\mu)$ is AL-Sup-δ-ß-comp-(*resp*. AL-Sup-δ-ß-Linde).

Proof: Assume $\mathcal{K}$ is Sup-δ-ß-clopen subset of $\mathcal{X}$ and $\{\mathcal{D}_i: i \in \mho\}$ is Sup-δ-ß-open-cover of $\mathcal{K}$. In that case $\mathcal{K}^c$ is Sup-δ-ß-clopen and $\mathcal{K} \subseteq \bigcup_{i\in\mho} \mathcal{D}_i$. Consequently, $\mathcal{X} = \bigcup_{i\in\mho} \mathcal{D}_i \cup \mathcal{K}^c$. As $\mathcal{X}$ is AL-Sup-δ-ß-comp, so $\mathcal{X} = \bigcup_{i=1}^{i=m} \delta\beta Cl^{\mu}(\mathcal{D}_i) \cup \mathcal{K}^c$. Hence, $\mathcal{K} \subseteq \bigcup_{i=1}^{i=m} \delta\beta Cl^{\mu}(\mathcal{D}_i)$. So, $\mathcal{K}$ is AL-Sup-δ-ß-comp.

By the same technique, one can prove theorem in case of Sup-δ-ß-Linde-sp.

Theorem 4.15: Let $\mathcal{H}$ be AL-Sup-δ-ß-comp-(*resp*. AL-Sup-δ-ß-Linde) subset of $\mathcal{X}$ and $\mathcal{M}$ is a Sup-δ-ß-clopen subset of $\mathcal{X}$, so $\mathcal{M} \cap \mathcal{H}$ is AL-Sup-δ-ß-comp-(*resp*. AL-Sup-δ-ß-Linde).

Proof: Suppose $\nabla = \{\mathcal{D}_i: i \in \mho\}$ is AL-Sup-δ-ß-open-cover of $\mathcal{H} \cap \mathcal{M}$. In that case $\mathcal{H} \subseteq \bigcup_{i\in\mho} \mathcal{D}_i \cup \mathcal{M}^c$. As, $\mathcal{H}$ is AL-Sup-δ-ß-comp-sets, so $\mathcal{H} \subseteq \bigcup_{i=1}^{i=m} \delta\beta Cl^{\mu}(\mathcal{D}_i) \cup \mathcal{M}^c$.

Consequently, $\mathcal{H} \cap \mathcal{M} \subseteq \bigcup_{i=1}^{i=m} \delta\beta Cl^{\mu}(\mathcal{D}_i)$. Therefore $\mathcal{H} \cap \mathcal{M}$ is AL-Sup-δ-ß-comp.

The other case for AL-Sup-δ-ß-Linde-sp can be made similarly.

Theorem 4.16: The Sup-δ-ß-Cont image of AL-Sup-δ-ß-comp-(*resp*. AL-Sup-δ-ß-Linde) set is AL-comp-(*resp*. AL-δ-ß-Linde).

Proof: Presume $\mathcal{G}: \mathcal{X} \longrightarrow \mathcal{Y}$ is Sup-δ-ß-Cont map and $\mathcal{H}$ is AL-Sup-δ-ß-comp-subset of$\mathcal{X}$. Assume $\{\mathcal{D}_i: i \in \mho\}$ op-cover in $\mathcal{G}(\mathcal{H})$.

Now, $\mathcal{G}(\mathcal{H}) \subseteq \bigcup_{i\in\mho} \mathcal{D}_i \Longrightarrow \mathcal{H} \subseteq \bigcup_{i\in\mho} \mathcal{G}^{-1}(\mathcal{D}_i)$. As, $\mathcal{G}$ is Sup-δ-ß-Cont, in that case $\mathcal{G}^{-1}(\mathcal{D}_i)$ is Sup-δ-ß-open, $\forall\, i \in \mho$. Via supposition, $\mathcal{H}$ is AL-Sup-δ-ß-comp, consequently $\mathcal{H} \subseteq \bigcup_{i=1}^{i=m} \delta\beta Cl^{\mu}(\mathcal{G}^{-1}(\mathcal{D}_i))$. as, $\mathcal{G}$ is Sup-δ-ß-Cont, in that case $\delta\beta Cl^{\mu}(\mathcal{G}^{-1}(\mathcal{D}_i)) \subseteq \mathcal{G}^{-1}(Cl(\mathcal{D}_i))$.

Thus, $\mathcal{G}(\mathcal{H}) \subseteq \bigcup_{i=1}^{i=m} \mathcal{G}((\mathcal{G}^{-1}(Cl(\mathcal{D}_i)))) \subseteq \bigcup_{i=1}^{i=m} Cl(\mathcal{D}_i)$. Hence, $\mathcal{G}(\mathcal{H})$ is AL-comp.

The other case for almost Sup-δ-ß-Linde-sp can be made similarly.

Theorem 4.17: Let each family of Sup-δ-ß-closed subset of Sup-Top-Sp-$(\mathcal{X},\mu)$, satisfying the FIT, has itself, a non-empty intersection, so $\mathcal{X}$ is AL-Sup-δ-ß-comp.

Proof: Presume $\nabla = \{\mathcal{D}_i: i \in \mho\}$ is Sup-δ-ß-open-cover of $\mathcal{X}$. Assume $\{\mathcal{D}_i: i \in \mho\}$ has no finit sub-family which is the Sup-δ-ß-closures of whose members cover $\mathcal{X}$. In that case $\mathcal{X} \setminus \bigcup_{i=1}^{i=m} \delta\beta Cl^{\mu}(\mathcal{D}_i) \neq \emptyset, \forall m \in \mathbb{N}$. Consequently, $\mathcal{X} \setminus \bigcup_{i=1}^{i=m} (\mathcal{D}_i) \neq \emptyset$, Now, $\bigcap_{i=1}^{i=m} \mathcal{D}_i^c \neq \emptyset$ and this implies $\{\mathcal{D}_i^c: i \in \mho\}$ is family of Sup-δ-ß-closed subset of $\mathcal{X}$ which has the FIP. Hence, $\bigcap_{i\in\mho} \mathcal{D}_i^c \neq \emptyset$ and implies$\mathcal{X} \neq \bigcup_{i\in\mho} \mathcal{D}_i$. Except this contradicts ∇ is Sup-δ-ß-op-cover in $\mathcal{X}$. Therefore, $\mathcal{X}$ is AL-Sup-δ-ß-comp.

Remark 4.18: The next example illustrates the converse of the above Theorem is not always hold.

Example 4.19: The Sup-Top-Sp $(\mathbb{N},\boldsymbol{\mu})$ which described in Example 4.3 is AL-Sup-δ-ß-comp. A family of Sup-δ-ß-closed subset $\{\mathcal{H}_m: \mathcal{H}_m = \{2m + 2, 2m + 3, \ldots\}$ and $m \in \mathbb{N}\}$ of $(\mathbb{N},\mu)$ has FIP, while $\bigcap_{i=1}^{\infty} \mathcal{H}_m = \emptyset$.

5- Miscellaneous Results of Mildly Supra-*δ*-*ß*-Compact and Mildly Supra-*δ*-*ß*-Lindelöf spaces

In this segment, novel kinds of extended mildly supra compact (briefly, M-Sup-Comp) and mildly supra Lindelöf (briefly, M-Sup-Linde)-sp have been studied and various essential characterizations related to these kinds of extended M-Sup-compact and M-Sup-Lindelöf spaces have been investigated.

Definition 5.1: A Sup-Top-Sp $(\mathcal{X},\mu)$ is said to be M-Sup-*δ*-*ß*-comp-(*resp*. M-Sup-*δ*-*ß*-Linde) if each Sup-*δ*-*ß*-clopen-cover of $\mathcal{X}$has finite (*resp*. countable) sub-cover.

Definition 5.2: A subset $\mathcal{W} \subseteq \mathcal{X}$ is called M-Sup-*δ*-*ß*-comp-(*resp*. M-Sup-*δ*-*ß*-Linde) relative to $\mathcal{X}$ if each Sup-*δ*-*ß*-clopen-cover of $\mathcal{W}$ has finite (*resp*. countable) sub-cover.

Proposition5.3: Each M-Sup-*δ*-*ß*-comp-sp is M-Sup-*δ*-*ß*-Linde-sp.

Proof: It is proof is straightforward.

Remark 5.4: The subsequently example explain the reverse of Proposition 5.3, isn't always hold .

Example 5.5: If we change$\mathbb{R}$ via $\mathbb{Q}$ in Example 4.4, so $\nabla = \{\{1,p,q\}: p \neq q \neq 1 \in \mathbb{Q}\}$ is Sup-*δ*-*ß*-clopen-cover for $\mathbb{Q}$. Clearly, ∇ has countable-cover, while it hasn't finite sub-cover.

Theorem 5.6: Each M-Sup-*δ*-*ß*-comp-(*resp*. M-Sup-*δ*-*ß*-Linde)-sp $(\mathcal{X},\mu)$ is M-Sup-*δ*-*ß*-comp (*resp*. M-Sup-*δ*-*ß*-Linde).

Proof: Presume a collection $\nabla = \{\mathcal{A}_i : i \in \mho\}$ is Sup-*δ*-*ß*-clopen-cover of $(\mathcal{X},\mu)$. As $(\mathcal{X},\mu)$ is M-Sup-*δ*-*ß*-comp, in that case $\mathcal{X} = \bigcup_{i=1}^{i=m} \delta\beta Cl^{\mu}(\mathcal{A}_i)$ and because $\mathcal{A}_i$ is Sup-*δ*-*ß*-clopen set, so $\delta\beta Cl^{\mu}(\mathcal{A}_i) = \mathcal{A}_i$. Therefore, $(\mathcal{X},\mu)$ is M-Sup-*δ*-*ß*-comp.

The proof for M-*δ*-*ß*-Linde-sp is similar.

The other case for M-Sup-δ-ß-Lindelöf spaces can be made similarly.

The converse of the above Theorem is still an open problem.

Corollary 5.7: Each Sup-*δ*-*ß*-comp-(*resp*. Sup-*δ*-*ß*-Linde)-sp is M-Sup-*δ*-*ß*-comp-(*resp*. M-Sup-*δ*-*ß*-Linde).

Proof: It is straightforward and follows directly from their related definitions.

Remark 5.8: The converse of the above Corollary is not correct always as shown in the next interesting example :

Example 5.9: Presume $\mu = \{\emptyset, \mathbb{R}, \{1\}\}$ is Sup-Top on $\mathbb{R}$. Assume there exists Sup-*δ*-*ß*-clopen set $\mathcal{K}$ different $\emptyset$ & R. In that case, $Int^{\mu}\left(Cl^{\mu}\left(Int^{\mu}_{\delta}(\mathcal{K})\right)\right) \subseteq \mathcal{K} \subseteq Cl^{\mu}\left(Int^{\mu}\left(Cl^{\mu}_{\delta}(\mathcal{K})\right)\right)$. This implies $\{1\} \subseteq \mathcal{K}$. As a result, $Int^{\mu}\left(Cl^{\mu}\left(Int^{\mu}_{\delta}(\mathcal{K})\right)\right) = \mathbb{R}$.

This contradicts that $Int^{\mu}\left(Cl^{\mu}\left(Int^{\mu}_{\delta}(\mathcal{K})\right)\right) \subseteq \mathcal{K}$. Hence, the only Sup-*δ*-*ß*-Clopen subsets in $(\mathbb{R},\mu)$ are $\emptyset$ & $\mathbb{R}$. Thus,$(\mathbb{R},\mu)$ M-Sup-*δ*-*ß*-comp. On other hand, a *δ*-*ß*-open-cover $\{\{1,p\}: p \in \mathbb{R}\}$ of $\mathbb{R}$ has no coumtable sub-cover. Consequently, $(\mathbb{R},\mu)$ is not Sup-*δ*-*ß*-Linde.

Proposition 5.10: Every finite (resp. countable) Sup-Top-Sp-$(\mathcal{X},\mu)$ is M-Sup-*δ*-*ß*-comp-(*resp*. M-Sup-*δ*-*ß*-Linde).

Proof: It is straightforward obtained from Definition 5.1,with proposition 3.12.

Proposition5.11: Finite (*resp*. countable) union of M-Sup-*δ*-*ß*-comp-sub-sets of Sup-Top-Sp is M-Sup-*δ*-*ß*-comp-(*resp*. M-Sup-*δ*-*ß*-Linde).

Proof: It is straightforward got via Definition 5.1,with proposition 3.13.

Theorem 5.12: Each Sup-*δ*-*ß*-clopen subset $\mathcal{K}$ of M-Sup-*δ*-*ß*-comp-(*resp*. M-Sup-*δ*-*ß*-Linde) sp- $(\mathcal{X},\mu)$ is M-Sup-*δ*-*ß*-comp-(*resp*. M-Sup-*δ*-*ß*-Linde).

Proof: Assume $\mathcal{K}$ is a supra *δ*-*ß*-clopen subset of $\mathcal{X}$ and $\{\mathcal{D}_i : i \in \mho\}$ is Sup-*δ*-*ß*-clopen cover of $\mathcal{K}$. In that case, $\mathcal{K}^c$ is Sup-*δ*-*ß*-clopen and $\mathcal{K} \subseteq \bigcup_{i\in\mho} \mathcal{D}_i$. Consequently $\mathcal{X} = \bigcup_{i\in\mho} \mathcal{D}_i \cup \mathcal{K}^c$. As $\mathcal{X}$ is M-Sup-*δ*-*ß*-comp, so $\mathcal{X} = \bigcup_{i=1}^{i=m} \mathcal{D}_i \cup \mathcal{K}^c$. as a result $\mathcal{K} \subseteq \bigcup_{i=1}^{i=m} \mathcal{D}_i$. Therefore, $\mathcal{K}$ is M-Sup-*δ*-*ß*-comp.

The other case for mildly Sup-δ-ß-Linde-sp can be made similarly.

Theorem 5.13: Let $\mathcal{H}$ be M-Sup-*δ*-*ß*-comp-(*resp*. M-Sup-*δ*-*ß*-Linde)subset in $(\mathcal{X},\mu)$and $\mathcal{M}$ Sup-*δ*-*ß*-clopen subset in $(\mathcal{X},\mu)$, so $\mathcal{H} \cap \mathcal{M}$ is M-Sup-*δ*-*ß*-comp-(*resp*. M-Sup-*δ*-*ß*-Linde).

Proof: Presume $\nabla = \{\mathcal{D}_i : i \in \mho\}$ is Sup-*δ*-*ß*-clopen cover of $\mathcal{H} \cap \mathcal{M}$. In that case $\mathcal{H} \subseteq \bigcup_{i\in\mho} \mathcal{D}_i \cup \mathcal{M}^c$. Because, $\mathcal{H}$ is M-Sup-*δ*-*ß*-comp, so $\mathcal{H} \subseteq \bigcup_{i=1}^{i=m} \mathcal{D}_i \cup \mathcal{M}^c$. Consequently, $\mathcal{H} \cap \mathcal{M} \subseteq \bigcup_{i=1}^{i=m} \mathcal{D}_i$. as a result $\mathcal{H} \cap \mathcal{M}$ is M-Sup-*δ*-*ß*-comp.

The other case for mildly Sup-δ-ß-Linde-sp can be made similarly.

Theorem 5.14: The Sup-δ-*ß*-Cont image of M-Sup-δ-*ß*-comp-(*resp*. M-Sup-δ-*ß*-Linde) set is comp-(*resp*. Linde).
Proof: It is analogous of that Theorem 3.20, hence removed.

Conclusion

The classes of Sup-compact and Lindelöf spaces are at the present one of most vital and necessary classes not only in extension Sup-Top, but in other branches of mathematics. Since topological classes, supra compact and Lindelöf spaces are of somewhat dissimilar characters than most other significant characterization, such as the covering properties which are studied in general topology . So compactness considered one of the fundamental classes of paramount interest for topologists. Thus, in our article presented and investigated other extensions for the classical meaning of Sup-compact and Lindelöf spaces by employing another class of Sup-open set namely Sup-δ-*ß*-open. Various essential properties related to these kinds of Sup-compactness and Lindelöfness are verified. As well, some examples to explain the relationships among these classes are proposed. We anticipate that the discoveries in this article will aid scientists in improving the researches on extension supra topology in order to elevate a general framework for their practical implementations in each advanced branches of mathematics and other sciences .

Reference

[1] R. Staum, "The algebra of bounded continuous functions into non archimedean field," *Pacic Journal of Mathematics*, vol. 50, no, 1, pp. 169-185, 1974.

[2] P. h. Lambrinos, "On almost compact and nearly compact spaces," *Rendiconti del Circolo Matematico di Palermo*, vol. 24, pp. 14-18, 1975.

[3] M. E. Abd El-Monsef and A. M. Kozae, "Some generalized forms of compactness and closedness," *Delta Journal of Science*, vol. 9, no. 2, pp. 257-269, 1985.

[4] M. Ganster, "Some remarks on strongly compact spaces and semi-compact spaces," *Bulletin Malaysian Mathematical Society*, vol. 10, no. 2, pp. 67-70, 1987.

[5] J. Dontchev, M. Ganster and T. Noiri, "On p-closed spaces," *International Journal of Mathematics and Mathematical Sciences*, vol. no. 24, pp. 203-212, 2000.

[6] **A**. A. Farhan and A. M. F. AL-Jumaili, "Another Certain Forms of Soft Compact and Soft Closed Spaces," *2021 International Conference on Advanced Computer Applications*, ACA 2021, 2021, pp. 10-15.

[7] H. H. A. Al-Salmani, and A. M. F. AL-Jumaili, "Another certain version of almost soft e-compactness and e-Lindelofness via soft e -open sets," *AIP Conference Proceedings, 2nd International Conference of Mathematics, Applied Sciences, Information and Communication Technology*, vol. 2834, no. 1, 2023, pp. 080034.

[8] A. S. Mashhour, A. A. Allam, F. S. Mahmoud and F. H. Khedr, "On supra topological spaces," *Indian Journal of Pure and Applied Mathematics*, vol. 14, No. 4, pp. 502–510, 1983.

[9] T. M. Al-shami, "Some results related to supra topological spaces," *Journal of Advanced Studies in Topology*, vol. 7, no. 4, pp. 283-294, 2016.

[10] Z. K. Raheem, S.H. Jasem, "nc-Sets in Topological Space," *Mustansiriyah Journal of Pure and Applied Sciences*, vol. 2, no. 1, pp, 115-123, 2024.

[11] T. M. Al-shami, B. A. Asaad and M. A. EL-Gayar, "Various types of supra pre-compact and supra pre-Lindelof spaces," *Missouri Journal of Mathematical Sciences*, vol. 32, no. 1, pp. 1-20, 2020.

[12] H. Sadiq and M. A. Khalik, "Some Results on Fuzzy **ω**-Local Covering Dimension Function in Fuzzy Topological Space," *Mustansiriyah Journal of Pure and Applied Sciences*, vol. 1, no. 2, pp. 11-20, 2023.

[13] S. Sarmad and Y. Y. Yousif, "Supra rough membership relations and supra fuzzy digraphs on related topologies," *Iraqi Journal of Science*, Special Issue, pp. 28–34, 2020.

[14] S. H. Abdulwahid and A. M. F. AL. Jumaili, "On E_c-Continuous and δ-$ß_c$-Continuous Mappings in Topological Spaces Via E_c-open and δ-$ß_c$-open sets," *Iraqi Journal of Science*, vol. 63, no. 7, pp. 3120-3134, 2022.

[15] A. M. F. AL. Jumaili, “Other New Versions of Generalized Neutrosophic Connectedness and Compactness and Their Applications,” *Mathematics and Statistics*, vol. 12, no. 1, pp. 16–23, 2024.

[16] M. Abo-Elhamayel and T. M. Al-shami, “Supra homeomorphism in supra topological ordered spaces,” *Facta Universitatis, Series: Mathematics and Informatics*, vol. 31, no. 5, pp. 1091-1106, 2016.

[17] M. E. El-Shafei, M. Abo-Elhamayel and T. M. Al-shami, “Strong separation axioms in supra topological ordered spaces,” *mathematical Sciences Letters*, vol. 6, no. 3, pp. 271-277, 2017.

[18] M. E. El-Shafei, M. Abo-Elhamayel and T. M. Al-shami, “Generating ordered maps via supra topological ordered spaces,” *International Journal of Modern Mathematical Sciences*, vol. 15, no. 3, pp. 339-357, 2017.

[19] T. M. Al-shami, “On some maps in supra topological ordered spaces,” *Journal of New Theory*, vol. 20, pp. 76-92, 2018.

[20] B. A. Asaad, M. K. Tahat and T. M. Al-shami, “Supra b-maps via topological ordered spaces,” *European Journal of Pure and Applied Mathematics*, vol. 12, no. 3, pp. 1231-1247, 2019.

[21] E. Hatir and T. Noiri, “On *δ-ß*-continuous functions, Chaos,” *Solitons and Fractals*, vol. 42, no. 1, pp. 205-211, 2009.

[22] M. D. Rashinghania and R. S. Aggarwal, “Topology for post graduate of Indian university,” S. Chand and Company Ltd, 1987, New Delhi.