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RESEARCH ARTICLE - MATHEMATICS

JACOBSON-SMALL COMPRESSIBLE MODULE

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Abstract

For an R-module Q and a commutative ring (with identity) R. In this paper, we introduce the notion Jacobson-small compressible (for short J −Small compressible) An R −module Q is J −Small compressible if Q can be embedded, in each of its non-zero J-small sub-module. Equivalently, Q is a J −Small compressible, if there exist a monomorphism from C into N whenever $0 \neq N \ll_J Q$ as a generalization of small compressible and we study some of the important characteristics and results of this concept. We also introduced the concept of Jacobson-small uniform and provide the relationship between Jacobson-small uniform and J −Small compressible.

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Keywords: J-small submodule, Compressible module, J −Small compressible module, $J - S\ uniform$ module.

Introduction

Let Q be the left unitary $R - module$, and let R be a commutative ring with 1. The concept of J-small sub-modules was introduced and studied by Ali K. and Wasan K. in [1], a submodule N of Q is called Jacobson-small (for short J-small, denoted by $N \ll_J Q$) if whenever $Q = N + K, K \subseteq Q$ such that $J\left(\frac{Q}{K}\right) = \frac{Q}{K}$ implies $Q = K$.

J. M. Zelmanowitz coined the term "compressible module" in 1976. If an R −module Q can be embedded, in every non-zero sub-module, that is, if there is a monomorphism $f: C \longrightarrow N$ for any N that is a non-zero sub-module N of Q, then Q is said to be compressible [2].

In this paper we introduce the concept of J −Small compressible as a generalization of the small compressible module. Many results and properties are studied in this paper. we also provide the ideas of J-small uniform modules (An R −module Q is called J-small uniform if every nonzero J- small sub-module of Q is essential.) and examine the connection between J −Small compressible and J-small uniform modules.

1. PRELIMINARIES

<u>Definition (1.1):</u>

1) A sub-module N of an R −module Q is termed a small sub-module ($N \ll Q$), if for every $K \leq Q$ with $N + K = Q$ implies that $K = Q$ [3].
2) A submodule N of Q is called Jacobson-small (for short J-small, denoted by $N \ll_J Q$) if whenever $Q = N + K, K \subseteq Q$ such that $J\left(\frac{Q}{K}\right) = \frac{Q}{K}$ implies $Q = K$[1].
3) An $R -$ module Q is called small uniform if every nonzero small sub-module of Q is essential in$Q$[4]).
4) If Q embedded in each of its nonzero sub-module. That is for each non-zero sub-module N of Q $\exists$ a monomorphism $f : Q \longrightarrow N$. then $R -$ module Q is called compressible [2].

5) If Q embedded in each of its nonzero small sub-module, then $R-$module Q is called small compressible. Equivalently, Q is small compressible if $\exists$ a monomorphism $f: Q \longrightarrow N$ whenever $0 \neq N \ll Q$ [5]

Remark (1.2)[1]:

1) Every small submodule of any R–module Q is J–small . But the convers is not true in general. For instance in the Z-module Z_6, let $A = \{0, 3\}, B = \{0, 2, 4\}$, $A + B = Z_6$, with $J\left(\frac{Z_6}{B}\right) = \frac{Z_6}{B}$. Thus A is J–small submodule Z_6 of but not small in Z_6.
2) Z –module Z_4, by (1) , {0} and { 0 , 2} are J–small in Z_4.
3) Consider $Q = Z \oplus Z_p^\infty$ as Z –module, since $\frac{Q}{Z} \cong Z_p^\infty$, then $J\left(\frac{Q}{Z}\right) \cong J(Z_p^\infty) = Z_p^\infty \cong \frac{Q}{Z}, J\left(\frac{Q}{Z}\right) = \left(\frac{Q}{Z}\right)$, but $Q \neq Z$. Implies that Z_p^∞ is not J–small $in\ Q$.

The following lemmas give as some properties of J-small submodules which will be used in this paper

Lemma (1.3)[1]:

1. Let Q be an R-module. If $J(Q) = Q$ and $A \subseteq Q$. Then $A \ll_J Q$ if and only if $A \ll Q$.
2. Let $f: Q \longrightarrow N$ be homomorphism. If $A \ll_J Q$ then $f(A) \ll_J N$.
3. Let $A \subseteq B \subseteq Q$, then $B \ll_J Q$ if and only if $\frac{B}{A} \ll_J \frac{Q}{A}$ and $A \ll_J Q$
4. Let $A \subseteq B$ be submodule of Q. If $A \ll_J B$, then $A \ll_J Q$
5. Let $Q = Q_1 \oplus Q_2$ be R-module and let $A_1 \subseteq Q_1$ and $A_2 \subseteq Q_2$. Then $A_1 \oplus A_2 \ll_J Q_1 \oplus Q_2$ if and only if $A_1 \ll_J Q_1$ and $A_2 \ll_J Q_2$.

2. JACOBSON-SMALL UNIFORM MODULES

In the section we introduce a generalization of uniform module which is called Jacobson-small uniform.

Definition (2.1):

An $R-$module Q is called J-small uniform(for short $J-S-uniform$) if every nonzero J-small sub-module of Q is essential.

Remarks(2.2):

Every uniform module is $J-S-uniform$. In general, the converse of (3) is not true, for instance, Z_6 as Z_6-module is $J-S-uniform$ (by logic) since it has no J-small sub-module. But it is not uniform, since $\langle \overline{3} \rangle, \langle \overline{2} \rangle \neq 0$ but not essential in Z_6.

Proposition (2.3)[3]

let Q be an R-module and let $N \leq K \leq Q$ then $N \leq_e K \leq_e Q$ if and only if $N \leq_e Q$.

Proposition (2.4):

Every nonzero sub-module of $J-S-uniform\ R-$module is $J-S-uniform$.

Proof:

Let Q be $J-S-uniform$, $0 \neq N \leq Q$ and $0 \neq K \ll_J N$. By lemma (1.3,4), $K \ll_J Q$ and by assumption $K \leq_e Q$. By Proposition(2.3),thus $K \leq_e N$.

Remark (2.5):

The direct-sum of $J-S-uniform$ modules, need not be $J-S-uniform\ R$-module, for instant each of the Z-modules Z_8 and Z_2 are $J-S-uniform$ (since they are uniform), but $Z_2 \oplus Z_8$ is not $J-S-uniform$ since $A = \{(\overline{1}, \overline{0})\}$ is nonzero J-small of $Z_2 \oplus Z_8$ but not essential in $Z_2 \oplus Z_8$.

Proposition (2.6):

Let Q be a $R-$module and $J(Q) = Q$. Then Q is $J-S-uniform$ if and only if Q is small uniform .

Proof:

$(\Longrightarrow)$ let $N \ll Q$, thus by Lemma (1.3,1), $0 \neq N \ll_J Q$, so by assumption $N \leq_e Q$. Therefore Q is $J-S-uniform$.

$(\Longleftarrow)$ By Lemma (1.3,1) we have every J-small sub-module is small, so we get desired result.

Proposition (2.7)[3]:

Let Q and Q' be R-module and let $f : Q \longrightarrow Q'$ be R-homomorphism. If $N \leq_e Q'$, then $f^{-1}(N) \leq_e Q$.

Proposition (2.8):

Let $f : Q_1 \longrightarrow Q_2$ be a module isomorphism. If Q_2 is $J-S-uniform$, then so Q_1 $J-S-uniform$.

Proof:

Let $0 \neq N \ll_J Q_1$, thus by lemma (1.3, 2) $f(N) \ll_J Q_2$. But Q_2 is $J-S-uniform$ so $f(N) \leq_e Q_2$ and by Proposition (2.7) $N \leq_e Q_1$. Therefore is Q_1 is $J-S-uniform$.

Recall that "An $R-$ module Q is called a multiplication module, if for each sub-module N of Q $\exists$ an ideal I of R such that $N = IQ$. In Q is called a multiplication module. if $[\,N{:}\,Q]Q = N$, for each sub-module N of Q[6].

An $R-$module Q is called faithful if $ann_R\,(Q) = 0$ [3].

Proposition (2.9)[7]:

Let Q be a finitely generated, faithful multiplication $R-$ module $N = QI \leq Q$. Then $N \leq_e Q$ if and only if $I \leq_e R$.

Proposition (2.10):

Let Q be a finitely generated faithful multiplication $R-$module. Let $N \leq Q$ and $(N{:}\,Q) \ll_J R$ then $N \ll_J Q$.

Proof:

Suppose that $N + K = Q, K \subseteq Q$ such that $J\left(\frac{Q}{K}\right) = \frac{Q}{K}$ and we want to prove that $K = Q$. Since Q is multiplication then $N = (N{:}\,Q)Q$ and $K = (K{:}\,Q)Q$. Thus $(N{:}\,Q)Q + (K{:}\,Q)Q = Q$, and since Q is finitely generated faithful multiplication module we have $(N{:}\,Q) + (K{:}\,Q) = R$ [7]. And since $(N{:}\,Q) \ll_J R$, then $R = (K{:}\,Q)$. Hence $Q = (K{:}\,Q)Q = K$ and therefore $N \ll_J Q$.

Proposition (2.11):

Let Q be a finitely generated, faithful multiplication $R-$module. If Q is $J-S-uniform$, then R is $J-S-uniform$ $R-$module.

Proof:

let $0 \neq I \ll_J R$, put $N = QI$. Then $0 \neq N \leq Q$, then by proposition (2.10) $N \ll_J Q$. But Q is $J-S-uniform$ so $N \leq_e Q$. Thus by Proposition (2.9) $I \leq_e R$, then R is $J-S-uniform$.

Recall that (An R-module Q is called due, if every sub-module of Q is fully invariant)[8].

Proposition (2.12)[9]:

Let $Q = \oplus_i Q_i$ for $(i \in I), R-$mod, where Q_i is a sub-module of Q $\forall$ $(i \in I)$, such that if $A_i \subseteq_e Q_i, \forall\ (i \in I)$ then $\oplus_i A_i \subseteq_e Q_i$.

Proposition (2.13):

Let Q_1 and Q_2 be R −module, such that $Q = Q_1 \oplus Q_2$ is duo module and for each $0 \neq N \ll_J Q$, $N \cap Q_1 \neq 0$, $N \cap Q_2 \neq 0$. If Q_1 and Q_2 are J-small uniform , then Q is $J - S - uniform.$

Proof:

Let $0 \neq N \ll_J Q$. Then $N = (N \cap Q_1) \oplus (N \cap Q_2)$[8][10]. As $N \cap Q_2 \leq_e Q_2$[7]. By (Proposition(2.12)), Thus $(N \cap Q_1) \oplus (N \cap Q_2) \leq_e Q$. Thus Q is $J - S - uniform.$

3. JACOBSON-SMALL COMPRESSIBLE MODULE

In the section, a generalization of small compressible module was given and studied

Definition(3.1):

An $R -$ module Q is called Jacobson-small compressible (for short J −Small compressible) , if Q can be embedded, in each of its non-zero J-small sub-module.

Equivalently, Q is a J −Small compressible, if there exist a monomorphism from Q into N whenever $0 \neq N \ll_J Q$.

If the R −module R is$J$ −Small compressible that is R can be embedded in any of its non-zero J-small ideal then, A Ring R is called J −Small compressible.

Examples and remarks(3.2):

1) Every J −Small compressible module is small compressible module.

Proof:

Let Q be J −Small compressible and let N be non zero small sub-module of Q. By remark (1.2) , every small sub module of any R −module Q is J-small, N is J- small sub-module. Since Q is J −Small compressible module, so there exist a monomorphism, $f: Q \longrightarrow N$. Thus Q small compressible module.

2) In general, the converse of (1) is not true, for instant Z_6 as Z-module is small compressible since $\langle \bar{0} \rangle$ is the only small sub-module of Z_6 but it is not J −Small compressible, since Z_6 have two non-zero J-small sub-module which are $\langle \bar{2} \rangle$, $\langle \bar{3} \rangle$, but Z_6can't be embedded in $\langle \bar{2} \rangle$ or in $\langle \bar{3} \rangle$.
3) Every compressible module is J −Small compressible.

Proof: clear

4) In general, the converse of (3) is not true. For instance, Z_6 as Z_6-module is J −Small compressible since the only J-small sub-module is $\langle \bar{0} \rangle$. But it is not compressible module [2].
5) The Z as Z-module is J −Small compressible.

Solution

Because its compressible and by (3), then Z as Z-module is J −Small compressible .

6) The Z_4 as Z-module is not J −Small compressible module since $\langle \bar{2} \rangle \ll_J Z_4$, but Z_4 cannot be embedded in $\langle \bar{2} \rangle$.
7) The Q as Z-module is not J −Small compressible since $Hom(Q, Z) = 0$.
8) Every simple module is J −Small compressible.

Proof:

Let Q be a simple R −module so Q have only two sub-module Q and (0). So Q is J −Small compressible (by logic).

9) The opposite of (8) is not true; for instance, the Z as Z-module is a J −Small compressible module, but it is not simple.

The following show that the converse of (1) in Remarks and examples (3.2) is true under certain condition.

proposition (3.3):

Let Q be a R −module and $J(Q) = Q$ then Q is J −Small compressible module, if and only if Q is small compressible module.

Proof:

($\Longrightarrow$) By (Remarks and examples$(3.2), 1$)

($\Longleftarrow$) Let Q be small compressible module and let $0 \neq N \ll_J Q$ by lemma(1.3) $N \ll Q$. But Q is small compressible so there exist a monomorphism, $f: Q \longrightarrow N$. Thus Q is J −Small compressible module.

Proposition (3.4):

An $R -$ module Q is J −Small compressible if and only if Q can be embedded in Rx for each $0 \neq x \in Q$ and $Rx \ll_J Q$.

Proof:

($\Longrightarrow$) by definition (3.1)

($\Longleftarrow$) Let $0 \neq N \ll_J Q$ and $0 \neq x \in Q$. Thus $Rx \ll_J Q$ by lemma (1.3) so by assumption there is a monomorphism $f: Q \longrightarrow Rx$. Thus the composition iof: $Q \longrightarrow N$ is a monomorphism where i: $Rx \longrightarrow N$ is the inclusion homomorphism. There Q is J −Small compressible.

The following proposition gives the converse of (3) from (Remarks and examples(3.2))

Proposition (3.5):

Let Q be an R −module in which every cyclic sub-module of Q is J-small in Q. Then Q is compressible, if and only if Q is J −Small compressible.

Proof:

($\Longrightarrow$) by (Remarks and examples$(3.2)(3)$)

($\Longleftarrow$) Let $0 \neq N \leq Q$ and $0 \neq x \in N$. Then by hypothesis $Rx \ll_J Q$. Again by hypothesis there exist a monomorphism, $f: Q \longrightarrow Rx$ and Thus the composition iof: $Q \longrightarrow N$ is where i: $Rx \longrightarrow N$ is the inclusion homomorphism. Therefore Q is compressible module.

Proposition (3.6):

Let Q be an R −module in which every sub-module of Q contains a nonzero J-small sub-module of Q. Then Q is compressible, if and only if Q is J −Small compressible.

Proof:

($\Longrightarrow$)by (Remarks and examples$(3.2)(3)$)

($\Longleftarrow$) Let Q be is J −Small compressible$R$ −module and let $0 \neq N \leq Q$. By hypothesis N contain a nonzero J-small sub-module of Q. Also by hypothesis there exist a monomorphism, $f: Q \longrightarrow L$. thus $iof: Q \longrightarrow N$ is a monomorphism and thus Q is compressible module.

Proposition(3.7):

A J-small sub-module of a J −Small compressible module is also J −Small compressible.

Proof:

Let Q be a is J −Small compressible module and $0 \neq N \ll_J Q$. To prove that N is J −Small compressible. Let $L \ll_J N$, by lemma (1.3) thus $L \ll_J Q$. Since Q is J −Small compressible module, there exist a monomorphism, $f: Q \longrightarrow L$. Hence $f \circ i: N \longrightarrow L$ is a monomorphism. Therefore, N is J −Small compressible module.

Corollary (3.8):

Every small sub-module of a J −Small compressible module is also J −Small compressible.

Proof:

Clear by remark (1.2) every small submodule of any R-module is J-small, and by proposition (3.7) we get the required result.

Proposition (3.9):

Every nonzero direct-summand, of a $J-S-compressible$ module is also $J-S-compressible$ module.

Proof:

Let Q be J −Small compressible module and let A_1 be nonzero direct-summand of Q. Thus $Q = A_1 \oplus A_2$ where $A_2 \leq Q$, let $0 \neq K \ll_J A_1$. By lemma (1.3) then $K \oplus (0) \ll_J Q$ and hence there is a monomorphism, $\emptyset: Q \to K \oplus (0)$. It's clear that $K \oplus (0) \cong K$. Thus $\emptyset: Q \to K$ is a monomorphism. Therefore the composition $\emptyset o j_{A_1} : A_1 \longrightarrow K$ is a monomorphism, where $j_{A_1} : A_1 \longrightarrow Q$ is the injective map of A_1 into Q. Thus A_1 is J −Small compressible module.

Remark (3.10):

In general, a homomorphic image of a J −Small compressible module need not J −Small compressible module. For instance: Z as Z-module is J −Small compressible module, but $\frac{Z}{4Z} \simeq Z_4$ is not J −Small compressible module.

Before giving a new proposition we need proof the following proposition :-

Proposition (3.11):

Let $f : Q \longrightarrow Q'$ be an epimorphism. If $N' \ll_J Q'$ then $f^{-1}(N') \ll_J Q$

Proof:

Suppose $f^{-1}(N') + A = Q$ with $J\left(\frac{Q}{A}\right) = \frac{Q}{A}$, where $A \leq Q$. Thus $N' + f(A) = f(Q)$, with $f\left(J\left(\frac{Q}{A}\right)\right) = f\left(\frac{Q}{A}\right) = \frac{Q'}{A}$ so $N' + f(A) = Q'$ with $f\left(\frac{Q}{A}\right) = \frac{Q'}{A}$. But $N' \ll_J Q'$ so $Q' = f(A)$ and hence $A = f^{-1}(Q')$. Therefore $f^{-1}(N') \ll_J Q$.

Proposition (3.12):

Let Q_1 and Q_2 are two R −module such that $Q_1 \cong Q_2$. Then Q_1 is J −Small compressible modules if and only if, Q_2 is J −Small compressible modules.

Proof:

Suppose that Q_1 is J −Small compressible module and $Q_1 \cong Q_2$. Let $\psi: Q_1 \longrightarrow Q_2$ be an isomorphism and $\mathrm{N} \ll_{\mathrm{J}} Q_2$. Then $0 \neq \psi^{-1}(N) \ll_{\mathrm{J}} Q_1$ by proposition (3.11). Put $\mathrm{L} = \psi^{-1}(N)$, by hypothesis there are a monomorphism, $g: Q_1 \longrightarrow \mathrm{L}$. let $f = \psi|_L$ then $f : \mathrm{L} \longrightarrow Q_2$ is a monomorphism, and $f(L) = \psi(\psi^{-1}(\mathrm{N}))$, thus, $f : \mathrm{L} \longrightarrow N$ is a monomorphism. Then the composition $Q_2 \xrightarrow{\psi^{-1}} Q_1 \xrightarrow{g} L \xrightarrow{f} N$. Put $h = fogo\psi^{-1}$. Thus Q_2 is J −Small compressible module.

Proposition (3.13):

Let $Q = Q_1 \oplus Q_2$ be an R −module such that $ann_R Q_1 + ann_R Q_2 = R$. Then Q is J −Small compressible module if and only if Q_1 and Q_2 are J −Small compressible module.

Proof:

($\Longrightarrow$) Clear by proposition (3.9)

($\Longleftarrow$) to prove $Q = Q_1 \oplus Q_2$ is J −Small compressible module, Let $0 \neq N \ll_{\mathrm{J}} Q$. Thus by [11], $N = K_1 \oplus K_2$, for some $0 \neq K_1 \leq Q_1 \leq Q$ and $0 \neq K_2 \leq Q_2 \leq Q$. Since $N \ll_J Q$ so by lemma (1.3) we have $K_1 \ll_{\mathrm{J}} Q_1$ and $K_2 \ll_J Q_2$. But by assumption Q_1 and Q_2 are J −Small compressible , so there are monomorphisms, $f: Q_1 \longrightarrow K_1$ and $g: Q_2 \longrightarrow K_2$. Now, define $h: Q \longrightarrow N$ by $h(x, y) = (f(x), g(y))$, it is clear that h is a monomorphism, and hence Q is J −Small compressible module.

Corollary(3.14):

Let $\{Q_i\}_{i=1}^{n}$ be a finite of $J-$ Small compressible $R-$module such that $\sum_{i=1}^{n} ann\ Q_1 = R$. Then $\oplus_{i=1}^{n} Q_i$ is also $J-$Small compressible $R-$module.

The following proposition to show that relation between $J-$ Small compressible $R-$module and $J-S-uniform$.

Proposition (3.15):

If Q is $J-$ Small compressible $R-$module, then Q is $J-S-uniform$.

Proof:

To prove that Q is $J-S-uniform$, let $0 \neq L \ll_J Q$, so let $0 \neq x \in L$. By lemma (1.3) $Rx \ll_J Q$. But is Q $J-$Small compressible so $\exists$ a monomorphism $f: Q \longrightarrow Rx$. Thus $f(x) = rx \neq 0$, for some $r \in R$, $r \neq 0$. Now, let $0 \neq a \in Q$ and $f(a) = tx,$ for some $t \in R$. As f is monomorphism, thus $tx \neq 0$. So $f(ra) = rf(a) = rtx = trx = tf(x) = f(tx)$. Since f is monomorphism $ra = tx$. But $tx \neq 0$, so $ra \neq 0$ and $ra \in N$, that is $N \leq_e Q$ and hence Q $J-S-uniform$.

Remark (3.16):

In general, the converse of (Proposition(3.15)) is not true, for instance the Z-module Z_4 is $J-S-uniform$, but is not $J-$Small compressible module since $\langle\bar{2}\rangle \ll_J Z_4$, but cannot be embedded, in $\langle\bar{2}\rangle$.

Corollary (3.17):

Every compressible is $J-S-uniform$.

Corollary (3.18):

Every simple module is $J-S-uniform$.

CONCLUSION

In this paper we introduced the concepts of Jacobson-Small Compressible and Jacobson-Small Uniform modules and we showed that it is an expand to the concept of Small Compressible modules and Small Uniform respectively . And we presented some important relations between Jacobson Small Compressible modules and Jacobson Uniform modules.

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